Math 2550 - HW 6 - Part 1
Vector spaces

---

1. Let $V = \mathbb{R}^3$ and $F = \mathbb{R}$. We talked about how V is a vector space over F in class using the usual vector addition and scalar multiplication:

$$\langle x, y, z\rangle + \langle a, b, c\rangle = \langle x + a, y + b, z + c\rangle$$

$$\alpha \cdot \langle x, y, z\rangle = \langle \alpha x, \alpha y, \alpha z\rangle$$

In this exercise you will verify some of the parts of the vector space definition.

   (a) List four elements from V and four elements from F.

   (b) Verify part 3 of the vector space definition: If $\vec{v}$ and $\vec{w}$ are in V, then $\vec{v} + \vec{w} = \vec{w} + \vec{v}$.

   (c) Verify part 9 of the vector space definition: If α is a scalar in F and $\vec{v}$ and $\vec{w}$ are vectors in V, then $\alpha \cdot (\vec{v} + \vec{w}) = \alpha \cdot \vec{w} + \alpha \cdot \vec{v}$.

---

2. Let $V = P_2$ and $F = \mathbb{R}$. Recall that

$$P_2 = \{a + bx + cx^2 \mid a, b, c \text{ are real numbers } \}$$

We talked about how V is a vector space over F in class using the usual polynomial addition and scalar multiplication:

$$(a_0 + b_0 x + c_0 x^2) + (a_1 + b_1 x + c_1 x^2) = (a_0 + a_1) + (b_0 + b_1)x + (c_0 + c_1)x^2$$

$$\alpha \cdot (a + bx + cx^2) = \alpha a + \alpha b x + \alpha c x^2$$

In this exercise you will verify some of the parts of the vector space definition.

   (a) List four elements from V and four elements from F.

   (b) Verify part 4 of the vector space definition: If $\vec{v}$, $\vec{w}$, $\vec{z}$ are in V, then $(\vec{v} + \vec{w}) + \vec{z} = \vec{v} + (\vec{w} + \vec{z})$.

   (c) Verify part 8 of the vector space definition: If α and β are scalars in F and $\vec{v}$ is a vector in V, then $(\alpha\beta) \cdot \vec{v} = \alpha \cdot (\beta \cdot \vec{v})$.

3. Let

$$V = \left\{ \begin{pmatrix} a & 1 \\ 1 & b \end{pmatrix} \;\middle|\; a \text{ and } b \text{ are real numbers} \right\}$$

and $F = \mathbb{R}$.

For addition we will use the usual matrix addition:

$$\begin{pmatrix} c & d \\ e & f \end{pmatrix} + \begin{pmatrix} g & h \\ i & j \end{pmatrix} = \begin{pmatrix} c+g & d+h \\ e+i & f+j \end{pmatrix}$$

For scalar multiplication we use the usual scalar multiplication:

$$\alpha \cdot \begin{pmatrix} c & d \\ e & f \end{pmatrix} = \begin{pmatrix} \alpha c & \alpha d \\ \alpha e & \alpha f \end{pmatrix}$$

(a) List out four elements from the set of vectors V.

(b) Show that V is not a vector space over F using the addition and scalar multiplication above.

[Hint: Do this by showing that V doesn't satisfy part 1 of the definition of a vector space. Recall that part 1 says: if $\vec{v}$ and $\vec{w}$ are in V, then $\vec{v} + \vec{w}$ is in V.]

---

4. Let $V = \mathbb{R}^2$ and $F = \mathbb{R}$.

For addition use the usual vector addition:

$$\langle x, y \rangle + \langle a, b \rangle = \langle x + a, y + b \rangle$$

Define a new scalar multiplication as follows:

$$\alpha \cdot \langle x, y \rangle = \langle 2\alpha x, 2\alpha y \rangle$$

(a) List out four elements from the set of vectors V.

(b) Compute $3 \cdot \langle 1, -2 \rangle$ using the scalar multiplication given above.

(c) Show that V is not a vector space over F using the addition and scalar multiplication above.

[Hint: Do this by showing that V doesn't satisfy part 7 of the definition of a vector space. Recall that part 7 says: if α is a scalar in F and $\vec{v}$ is in V then $1 \cdot \vec{v} = \vec{v}$.]