Math 455

Homework # 7 - Direct Products

1. Consider the group $G = \mathbb{Z}_2 \times \mathbb{Z}_3$. Compute the orders of each of the elements of G. Is G cyclic?

2. Find the order of $(\overline{2}, \overline{3})$ in $\mathbb{Z}_4 \times \mathbb{Z}_{12}$.

3. Let G be a group of order 4. Prove that either G is isomorphic to $\mathbb{Z}_4$ or G is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$.

4. Find all the subgroups of $\mathbb{Z}_2 \times \mathbb{Z}_4$ of order 4.

5. Let G be a group. Prove that if $G \times G$ is cyclic, then G is cyclic.

6. Let G and H be groups. Prove that if G and H are both abelian, then $G \times H$ is abelian.

7. Let G_1, G_2, H_1 and H_2 be groups. Prove that if G_1 is isomorphic to G_2 and H_1 is isomorphic to H_2, then $G_1 \times H_1$ is isomorphic to $G_2 \times H_2$.