Math 465 - Homework # 7
Uniform Continuity

1. (a) Prove that $f(x) = \dfrac{1}{x}$ is uniformly continuous on $[1, \infty)$.

   (b) Prove that $f(x) = \dfrac{1}{x}$ is not uniformly continuous on $(0, 1)$.

   (c) Prove that $f(x) = \dfrac{1}{1+x^2}$ is uniformly continuous on all of $\mathbb{R}$.

2. Show that if $f : D \to \mathbb{R}$ and $g : D \to \mathbb{R}$ are both uniformly continuous on D then $f + g$ is uniformly continuous on D.

3. Suppose that $f : D \to \mathbb{R}$ and $g : D \to \mathbb{R}$ are both uniformly continuous on D. Prove that if f and g are both bounded on D, then $f \cdot g$ is uniformly continuous on D.

4. Suppose that $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are both uniformly continuous on $\mathbb{R}$. Prove that $f \circ g$ is uniformly continuous on $\mathbb{R}$.

5. Suppose that $f : D \to \mathbb{R}$ is uniformly continuous on D and that (x_n) is a Cauchy sequence with $x_n \in D$ for all n. Prove that $(f(x_n))$ is a Cauchy sequence.