

Math 5680

Homework # 7

Identity Theorem

1. Let $f : A \rightarrow \mathbb{C}$ be analytic on a region A . Let $z_1, z_2 \in A$. Suppose that $f'(z_1) \neq 0$. Prove that f is not constant in any open disc D contained in A where $z_2 \in D$.
2. Let $U = \{z \mid |z| < 1 \text{ or } |z| > 2\}$ and let $f : U \rightarrow \mathbb{C}$ be defined by

$$f(z) = \begin{cases} z & , \text{ if } |z| < 1 \\ z^2 & , \text{ if } |z| > 2 \end{cases}$$

Determine if there exists an entire function that agrees with f on U .