Math 5680

Homework # 8

Rouche's Theorem

1. Show that all of the zeros of $p(z) = z^6 - 5z^2 + 10$ lie in the annulus

$$A = \{z \mid 1 \leq |z| < 2\}$$

2. Let n ber a positive integer and let c be a real number satisfying $c > e$. Show that the equation $e^z = cz^n$ has n solutions inside the unit circle.

   [An example of such an equation would be $e^z = 10z^{32}$]

3. A **fixed point** of a function g is a point z where $g(z) = z$.

   Let g be analytic inside and on the unit circle $|z| = 1$. Suppose that $0 < |g(z)| < 1$ if $|z| = 1$. Show that g has exactly one fixed point inside the unit circle.