Math 4680 - Homework # 9

Cauchy's Theorem

1. Evaluate the following integrals.

   (a) $\int_\gamma (z^2 - z + 10)\ dz$ where γ is the upper half of the unit circle oriented counterclockwise.

   (b) $\int_\gamma (z^2 - z + 10)$ where γ is the unit circle.

   (c) $\int_\gamma e^{1/z}\ dz$ where γ is a circle of radius 2 centered at $2 + i$.

   (d) $\int_\gamma \frac{1}{\sin(z)}\ dz$ where γ is the box with corners at $\frac{1}{2}$, $\frac{5}{2}$, $\frac{5}{2} + 3i$, and $\frac{1}{2} + 3i$. Orient γ in the counterclockwise direction.

   (e) $\int_\gamma z^i\ dz$ where γ is the curve composed of line segments from $1 - i$ to $1 + i$ to $-1 + i$ to -1. Here to define z^i choose the branch of the logarithm corresponding to $-\frac{\pi}{2} < \arg(z) < \frac{3\pi}{2}$.

2. Let γ_1 be the circle of radius 1 and let γ_2 be the circle of radius 2 (both oriented counterclockwise and centered at the origin). Show that

$$\int_{\gamma_1} \frac{dz}{z^{10}(z^2+9)}\ dz = \int_{\gamma_2} \frac{dz}{z^{10}(z^2+9)}\ dz$$