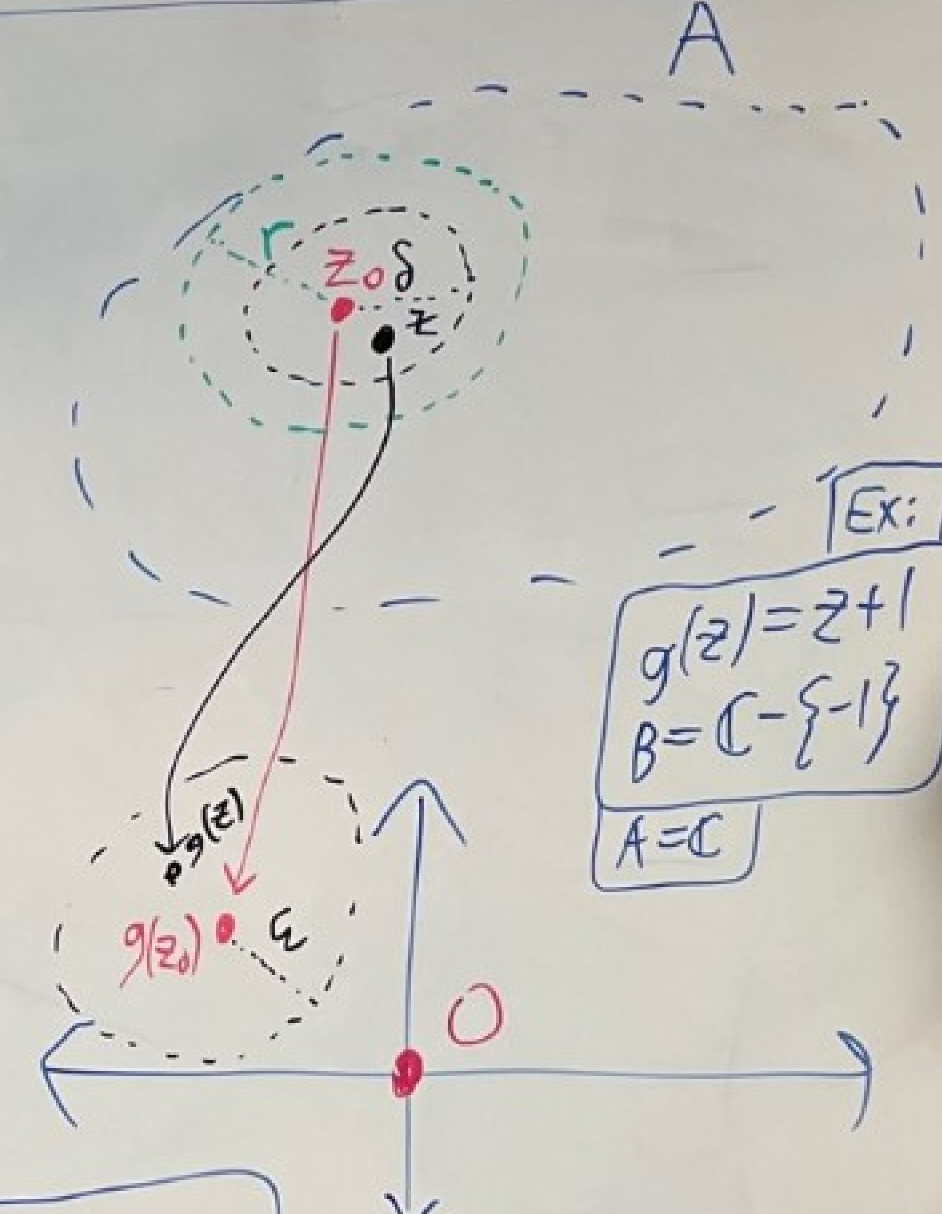
$\boxed{\text{HW 5}}$ ⑤ Let g be analytic on an open set A.
Let $B = \{z \in A \mid g(z) \neq 0\}$. Show (a) B is open, and (b) $\frac{1}{g(z)}$ is analytic on B.

(a) Since g is analytic on A, we know g is continuous on A.
Let $z_0 \in B$. Then $g(z_0) \neq 0$.
Since g is continuous at z_0 we know $\lim_{z \to z_0} g(z) = g(z_0)$.
Since $z_0 \in A$ and A is open, there exists $r > 0$ where $D(z_0; r) \subseteq A$.

Let $\varepsilon = \dfrac{|g(z_0) - 0|}{2} = \dfrac{|g(z_0)|}{2}$.

Since $\lim_{z \to z_0} g(z) = g(z_0)$ there exists $\delta > 0$ (make $\delta < r$)
where if $\underbrace{|z - z_0| < \delta}_{\text{same as: } z \in D(z_0; \delta)}$, then $\underbrace{|g(z) - g(z_0)| < \varepsilon}_{\text{same as: } g(z) \in D(g(z_0); \varepsilon)}$.

Let $z \in D(z_0; \delta)$. Why is $g(z) \neq 0$. If $g(z) = 0$, then
$|g(z_0)| = |g(z) - g(z_0)| < \varepsilon = \dfrac{|g(z_0)|}{2}$. Can't happen. Thus, $D(z_0; \delta) \subseteq B$
So, z_0 is an interior point and B is open. $\boxed{\text{(a)}}$



Ex:
$g(z) = z + 1$
$B = \mathbb{C} - \{-1\}$
$A = \mathbb{C}$