

Guided Pre-Class Practice

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Department of Mathematics, CSULA

Math 1040 Pre-Calculus – Fall 2019

*Instructor: Hillary Vo***Guided Pre-Class Practice****Topic:** Operations with Complex Numbers (Imaginary Numbers)**Date assigned:** 08/23/2019**Date due:** 08/30/2019**Time estimate to complete this assignment:** 45 minutes**Materials needed:** an account on MyOpenMath.com, any devices connected to the Internet, online videos (see Appendix 1), pre-class worksheets (see Appendix 2), pencil, and paper.**Overview/ Introduction****What is this lesson about?**

Introduce to Lesson: We have learned that not all quadratic equations have real-number solutions. For example, $x^2 = -1$ has no real-number solutions because the square of any real number x is never a negative number. To overcome this problem, mathematicians created an expanded system of numbers using the imaginary unit i , defined as $i = \sqrt{-1}$. Note that $i^2 = -1$.

This **guided pre-class practice** is about the basic ideas of complex numbers that students need to learn such as: the definition of complex numbers, the operations of complex numbers, complex conjugates, plotting complex numbers on a complex plan, and calculating absolute values of complex numbers.

Why do we care?

Background: Math 1040 course covers fundamental concepts in Pre-Calculus. By passing MATH 1040, students will fulfill their General Education B4 requirement. Then, students can keep continuing their higher education such as Calculus I, II, III, and so on. **Who this Pre-Calculus course is for:** MATH 1040 is intended primarily for students in the majors that require MATH 2110 (Calculus I), namely: Chemistry and Biochemistry (Biochemistry, B.S.: Bioinformatics and Computational Biology Minor, Chemistry, B.S.), Civil Engineering (Civil Engineering, B.S.), Computer Science (Computer Science, B.S.), Economics and Statistics (Economics, B.A), Electrical Engineering (Electrical Engineering, B.S.), ECST (Engineering with Special Options, B.S.), Geosciences and Environment (Geology, B.S., Geology Option: Geology, B.S., Environmental Geosciences Option), Kinesiology & Nutritional Science (Food Science & Technology, B.S.), Mathematics (Mathematics, B.S.), Mechanical Engineering (Mechanical Engineering, B.S.), Natural Science (Options I and II, not biology emphasis), Physics and Astronomy (all programs).

Thanks to the **Guided pre-class practice**, students will learn significant knowledge of complex numbers that is very useful for their future majors and is also one of the most important topics in any Pre-Calculus courses. Students are assigned to read the text-book on the course website www.MyOpenMath.com, watch the videos, and practice pre-class worksheets. Once finishing all assignments in the Guided pre-class practice, students will be able to understand the basic ideas of complex numbers. *Next step:* Students are able to apply their knowledge into advanced practice problems in-class and post-class activities later.

Learning Objectives

Basic Objectives for preparatory work

1. Give a definition and examples of complex numbers.
2. Give examples of complex conjugates.
3. Plot a point on a complex plane.
4. Practice adding and subtracting complex numbers.

Advanced objectives for classwork and after class work

1. Practice multiplying and dividing complex numbers.
2. Solve a quadratic equation that involves complex numbers.
3. Find the absolute values of complex numbers.
4. Use of complex numbers in real life.

Pre-class Individual Space Activities and Resources [45 minutes]

Procedure	Purpose	Estimated Time	Learning Objective
<i>Step 1:</i> Read Section 8.1 on www.MyOpenMath.com course website.	Introduce students to some basic ideas about complex numbers.	12 minutes	#1 (basic) through #4 (basic)
<i>Step 2:</i> Watch the video 1 “Introduction to Complex Numbers” (see Appendix 1), then answer question 1 (see Appendix 2): Give definition and three examples of complex numbers.	Provide students with the definition of complex numbers and give some examples of complex numbers.	7 minutes	#1 (basic)
<i>Step 3:</i> Answer question 2 (see Appendix 2): Express an imaginary number in standard form: a) $\sqrt{-16}$ b) $\sqrt{-20}$	Check on students’ understanding of writing a complex number in standard form of $a + bi$.	3 minutes	#1 (basic)
<i>Step 4:</i> Go over Section 8.1 again (if necessary), then answer question 3 (see Appendix 2): Give three examples of complex conjugates.	Introduce students to some basic ideas of complex conjugates.	4 minutes	#2 (basic)
<i>Step 5:</i> Watch the video 2 “Plot numbers on the complex plane” (see Appendix 1).	Describe how to plot a point on a complex plane.	3 minutes	#3 (basic)

<i>Step 6:</i> Answer question 4 (see Appendix 2): Plot this complex number on the complex plane: $3 - 4i$	Check on students' understanding of plotting a complex number on the complex plane.	3 minutes	#3 (basic)
<i>Step 7:</i> Watch the video 3 "Adding and Subtracting Complex Number".	Show students how to add and subtract complex numbers.	3 minutes	#4 (basic)
<i>Step 8:</i> Practice question 5 (see Appendix 2): a) Add $5 - 4i$ and $2 + 7i$ b) Subtract $2 + 5i$ from $3 - 6i$	Check on students' understanding of adding and subtracting complex numbers.	3 minutes	#4 (basic)
<i>Step 9:</i> Double check your answers before submitting your papers.	Build students confidence of their work and their carefulness in solving mathematics problems.	3 minutes	#1 (basic) through #4 (basic)
<i>Step 10:</i> Watch the Video 4: "Multiplying and Dividing Complex Numbers" and then practice question 6 (see Appendix 2).	Prerequisite for starting the advanced objectives for classwork and after classwork later.	4 minutes	#1 (advanced)

Appendix 1

Video Resources

Video 1: "Introduction to Complex Numbers"

https://www.youtube.com/watch?time_continue=191&v=NeTRNpBII7I

Video 2: "Plotting Numbers on the Complex Plane" <https://www.khanacademy.org/math/algebra2/introduction-to-complex-numbers-algebra-2/the-complex-plane-algebra-2/v/plotting-complex-numbers-on-the-complex-plane>

Video 3: "Adding and Subtracting Complex Numbers" <https://www.youtube.com/watch?v=SGhTjioGqqA>

Video 4: "Multiplying and Dividing Complex Numbers" <https://www.youtube.com/watch?v=O9xQali0NX0>

Appendix 2**Pre-class Activities**

Part A: (Question #1 to #5) Please complete this pre-class assignment by 12:00 pm, on Friday 08/30/2019, then submit it online through our course website (Math 1040/ Section 8.1 Complex Numbers):

- You can choose either to type your answers by selecting Text Entry box **or** to upload your papers under files of: .pdf, .doc, and/or .docx.
- If you do not have access to the Internet at your area, please let me know in advance and we will make alternative arrangements.
- This pre-class assignment will be graded for a total of 5 participation points.

Question 1 (1pts): Give definition and three examples of complex numbers.

Question 2 (1pts): Express the imaginary number in standard form.

- a) $\sqrt{-16}$
- b) $\sqrt{-20}$

Question 3 (1pts): Give three examples of complex conjugates.

Question 4 (1pts): Plot this complex number on the complex plane: $3 - 4i$

Question 5 (1pts): Add and subtract complex numbers.

- a) Add $5 - 4i$ and $2 + 7i$
- b) Subtract $2 + 5i$ from $3 - 6i$

Part B: (Entry ticket) Please practice and bring **question #6** to class for group discussion on 08/30/2019

Question 6: Multiply and divide complex numbers

- a) Multiply $(4 - 3i)(5 + 2i)$
- b) Divide $(2 + 5i)$ by $(3 - 2i)$

Question? If you have any questions, please come to my office hours (Monday and Wednesday 12:00 pm – 1:00 pm at ST-F213) **or** contact me via e-mail: hvo14@calstatela.edu.

Flipped IN-CLASS Lesson Plan

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Flipped IN-CLASS Lesson Plan

Topic: **Operations with Complex Numbers (Imaginary Numbers)**

Date: 08/30/2019

Time estimate to complete this assignment: 85 minutes

Materials needed: an account on MyOpenMath.com, any devices connected to the Internet, online videos (see Appendix 1), pre-class worksheets (see Appendix 2), in-class worksheets (see Appendix 3), pencil, and paper.

Learning Objectives	Basic Objectives for preparatory work	Advanced objectives for classwork and after classwork
1	Give a definition and examples of complex numbers.	Practice multiplying and dividing complex numbers.
2	Give examples complex conjugates.	Solve a quadratic equation involving complex numbers.
3	Plot a point on a complex plane.	Find the absolute values of complex numbers.
4	Practice adding and subtracting complex numbers.	Use of complex numbers in real life.

Flipped IN-CLASS Lesson Plan [85 minutes]

	Time Planned	Activity	Rationale	Resource Needed
<i>Beginning of class period</i>	6 minutes	<ul style="list-style-type: none"> - Review pre-class activities assignment (question 1 to 5 from Pre-class activities, see Appendix 2). - Collect student's entry tickets (question 6 from Pre-class activities, see Appendix 2). 	Check on students understanding about the basic objectives of learning complex numbers.	<ul style="list-style-type: none"> - Paper, pencil, and chalk. - All videos (see Appendix 1) - Problem sheets (see Appendix 2). - Section 8.1 (Complex Numbers).
<i>Middle of period 1</i>	8 minutes	<ul style="list-style-type: none"> - Mini-lecture based on Q&A from preparatory activities. - Ask and answer questions for any misconceptions. 	Make sure students understand well all basic objectives of complex numbers.	Course website: www.MyOpenMath.com

<i>Middle of period 2</i>	15 minutes	Introduce new lecture on multiply and divide complex numbers.	Clarify misconceptions and offer new information about complex numbers.	- Video 4 (see Appendix 1). - Course website: www.MyOpenMath.com
<i>Middle of period 3</i>	6 minutes	- Assign students working in group on question 6 . - Use think-pair-share strategy: students at first show their own work, then they can share and discuss with their partners.	Check on students understanding about multiplying and dividing complex numbers.	- Video 4 (see Appendix 1). - Question 6 (see Appendix 2). - Course website: www.MyOpenMath.com
<i>Middle of period 4</i>	10 minutes	- Introduce new practice problems on the board. - Continue using Think-pair-share technique: <ul style="list-style-type: none"> • Students make notes on their own, then share notes with their partners. • Each group need to show work on the board, then present their answers to the class. 	- Make sure each student involves in class activity. - Use physical movement on the board: it is one of the most useful methods to help students engage more in class. - Check on students understanding. - Clarify any misconceptions.	- Paper, pencil, and chalk. - Course website: www.MyOpenMath.com
<i>Middle of period 5</i>	12 minutes	- Continue explaining Advanced objectives # 2 and #3. - Practice homework problems.	Allow students to learn more knowledge of complex numbers.	- Paper, pencil, and chalk. - Course website: www.MyOpenMath.com
<i>Middle of period 6</i>	8 minutes	- Show more advanced examples. - Give students a chance to discuss and ask questions. - Manage students to self-work and share with their partners.	Help students to understand deeper by exploring more advanced examples and solving different kinds of quadratic equations involve in complex numbers.	- Paper, pencil, and chalk. - Lists of examples. - Course website: www.MyOpenMath.com
<i>Middle of period 7</i>	10 minutes	Finish the last Advanced objectives: the application	Great time for each student to explore the application of complex	- Paper, pencil, and chalk.

		of complex numbers in real life.	numbers in the real-life. For the reason: help students to engage more into the topic of complex numbers.	- Lists of examples of complex numbers using in the real life. - Course website: www.MyOpenMath.com
<i>End of class period</i>	10 minutes	- Recall all Advanced objectives of Complex numbers topic. - Manage students to self-work and share with their partners. - Provide a hand-out of four problems as an Exit-ticket (see Appendix 3).	- Great time to ask and answer questions before the lecture ends. - Correct any misunderstanding at the last minutes (if any). - Check on students understanding at the end of the class.	- Paper, pencil, and chalk. - Problem sheets. - Course website: www.MyOpenMath.com

Flipped AFTER CLASS Work Plan [Extra view beforehand]

Advanced Learning Objective	Activity and Rationale	Instructions to Students
1. Practice multiplying and dividing complex numbers. 2. Solve a quadratic equation involve in complex numbers. 3. Find the absolute values of complex numbers. 4. Use of complex numbers in real life.	<p>Instructors:</p> <ul style="list-style-type: none"> - Provide students homework. - Upload all homework online. <p>- Offer homework solutions when students complete it.</p> <p>Students:</p> <ul style="list-style-type: none"> - Review Section 8.1 on the course website www.MyOpenMath.com - Review all videos (see Appendix 1). - Write one summary sheet of paper (double pages) for Section 8.1. - Complete homework Section 8.1 online on the course website www.MyOpenMath.com It is a great opportunity for students to practice and check on their understanding. 	<ul style="list-style-type: none"> - Homework Section 8.1 is due online on the course website www.MyOpenMath.com - Need to complete homework Section 8.1 by Monday, 09/02/2019. - When finishing this homework assignment, students will be able to see the solutions.

Question? If you have any questions in regard to this assignment, here are my office hours (Monday and Wednesday 12:00 pm – 1:00 pm at ST-F213) **and** the e-mail: hvo14@calstatela.edu.

Appendix 3

Exit-ticket Activities

Please finish these problems and remember to turn them in to me before you leave.

This activity will count 8 points for participation.

Problem 1: Multiply and divide complex numbers

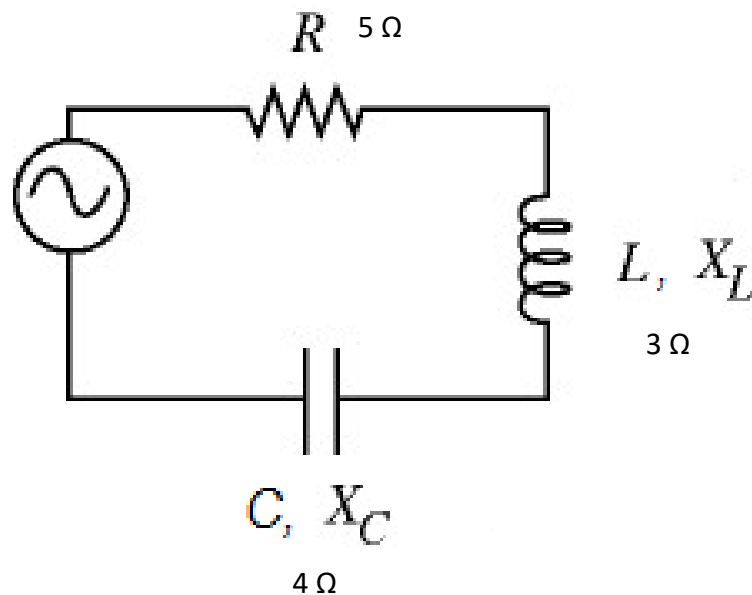
- a) Multiply $(7 - 5i)(4 + 6i)$
- b) Divide $(12 + 10i)$ by $(2 - 3i)$

Problem 2: Solve the quadratic equation $x^2 - 8 = -36$

Problem 3: Find absolute value of the complex number $-4 + 3i$

Problem 4: The application of complex numbers in a real word problem.

ELECTRICITY. Circuit components such as resistors, inductors, and capacitors all oppose the flow of current. This opposition is called resistance for resistors and reactance for inductors and capacitors. A circuit's total opposition to current flow is impedance. All of these quantities are measured in ohms (Ω). The impedance for a series circuit is the sum of the impedances for the individual components. Find the impedance of the circuit shown below:



Advanced Post-Class Practice

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Advanced Post-Class Practice

This is given for students to complete after the class meeting.

Topic: **Operations with Complex Numbers (Imaginary Numbers)**

Date assigned: 08/30/2019

Date due: 09/02/2019

Time estimate to complete this assignment: 120 minutes

Materials needed: an account on MyOpenMath.com, any devices connected to the Internet, online videos (see Appendix 1), post-class worksheets (see Appendix 4), pencil, and paper.

Post-class Activities and Deliverables:

Students will apply their knowledge of complex numbers to solve problems in Section 8.1 homework assignment (see Appendix 4). After completing this lesson, students will be able to:

- Give a definition and examples of complex numbers.
- Give examples of complex conjugates.
- Plot a point on a complex plane.
- Practice adding and subtracting complex numbers.
- Practice multiplying and dividing complex numbers.
- Solve a quadratic equation involve in complex numbers.
- Find the absolute values of complex numbers.
- Use of complex numbers in real life.

Here is the Flipped AFTER CLASS Work Plan [120 minutes]:

Advanced Learning Objectives	Activity and Rational	Time planned	Instructions to Students
1. Practice multiplying and dividing complex numbers. 2. Solve a quadratic equation involve in complex numbers. 3. Find the absolute values of complex numbers.	Instructor: On www.MyOpenMath.com : - Assign students some most important practice problems involve complex numbers. - Prepare and upload online homework for Section 8.1. - Provide homework solutions when students complete submission their homework.		- Homework Section 8.1 must be done online on the course website: www.MyOpenMath.com . - Need to follow carefully all instructions steps by steps. - Homework Due: on Monday, 09/02/2019 by 12:00pm. - Homework Solutions: when students finish submitting their homework assignments, they will be able to see the homework

4. Use of complex numbers in real life.	<p>Students:</p> <ul style="list-style-type: none"> - <i>Step 1:</i> Review Section 8.1 on the course website www.MyOpenMath.com - <i>Step 2:</i> Review all videos (see Appendix 1). - <i>Step 3:</i> Write one summary sheet of paper (double pages) for Section 8.1 Complex Numbers. - <i>Step 4:</i> Practice problems that involve complex numbers. - <i>Step 5:</i> Complete homework Section 8.1 online on the course website www.MyOpenMath.com 	120 minutes	<p>solutions under each homework question.</p> <p>Purpose:</p> <ul style="list-style-type: none"> ❖ It is really a great time for students to learn, review, practice problems, and do homework assignments that involve the complex numbers. ❖ Thanks to the homework solutions, students have a good opportunity to check on their understanding or any misconceptions about the complex numbers.
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Reflection Assignment: *About a flipped-learning practice with a topic of the complex numbers*

Please answer three following reflection questions:

1. How helpful do you think to study the concept of complex numbers by using a flipped-learning technique? Please justify your answer.
2. How do you perceive a flipped-classroom contributing to your ability to learn this math content and improve your critical thinking (Math 1040 Pre-Calculus – Section 8.1 Complex Numbers)?
3. What are your potential advantages and dis-advantages occurring while you were trying to finish pre-class, in-class, and post-class activities?

DUE: This reflection assignment will be due on Monday, 09/02/2019 by 12:00 pm on www.MyOpenMath.com (under Math 1040/ Students and Instructor Reflection Forum).

Connections to Future Lesson Plan(s): In this lesson plan, students have learned about the operations with complex numbers in rectangle planes. In future lesson plan (probably on Wednesday 11/13/2019), students will continue learning more topics of complex numbers in polar planes (for trigonometric part).

Question?

Any questions, please come to my office hours (Monday and Wednesday 12:00 pm – 1:00 pm at ST-F213) or contact me via e-mail: hvo14@calstatela.edu.

Note: If you do not have access to the Internet at your area, please let me know in advance and we will make alternative arrangements.

Appendix 4

Post-class Online Homework

Assignment 8.1: Complex Numbers

Learning Objectives: All Basic and Advanced Learning Objectives

1. (1 point)

Express $\sqrt{-36}$ as a complex number in standard form: $a + bi$ Answer: $0 + 6i$

2. (1 point)

Express $1 + \sqrt{-9}$ as a complex number in standard form: $a + bi$ Answer: $1 + 3i$

3. (1 point)

Perform the indicated operations & simplify, give your answer in standard form.

Add: $(10 - 5i) + (3 + 22i)$ Answer: $13 + 17i$ Subtract: $(10 - 5i) - (3 + 22i)$ Answer: $7 - 27i$

4. (1 point)

Express $\sqrt{-121} - \sqrt{-64}$ as a complex number in standard form:Answer: $3i$

5. (1 point)

Simplify the following expression $7(3 + 18i)$

If the answer is radical use sqrt (5) to denote (use the correct radicand in the problem!)

If the answer is complex use i to denote i .Answer: $21 + 126i$

6. (1 point)

Evaluate the expression $(2 + 4i)(-1 + 4i)$ and write the result in the form $a + bi$.Answer: $-18 + 4i$

7. (1 point)

Evaluate the expression $(2 + i)(-2 - i)$ and write the result in the form $a + bi$.**Hint:** Watch the following video “Multiplying Complex Numbers”<https://www.myopenmath.com/assessment/watchvid.php?url=http%3A%2F%2Fwww.youtube.com%2Fwatch%3Fv%3DFmr3o2zkwLM>Answer: $-3 - 4i$

8. (1 point)

What is the complex conjugate of the number $8 + 9i$?Answer: $8 - 9i$

9. (1 point)

The complex conjugate of $-2 - 18i$ is?

Answer: $-2 + 18i$

10. (1 point)

Evaluate the expression $\frac{4-3i}{2i}$ and write the result in standard form $a + bi$.

Hint: Watch the following video “Dividing Complex Numbers”

<https://www.myopenmath.com/assessment/watchvid.php?url=http%3A%2F%2Fwww.youtube.com%2Fwatch%3Fv%3DXBJjbJAwm1c>

Answer: $-\frac{3}{2} - 2i$

11. (1 point)

Simplify the following Expression $\frac{7+5i}{3+2i}$

Enter the answer as reduced fraction, when necessary.

Answer: $\frac{31}{13} + \frac{1}{13}i$

12. (1 point)

Simplify the following expression $\frac{1+7i}{7-6i}$

Enter the answer as reduced fraction, when necessary.

Answer: $-\frac{7}{17} + \frac{11}{17}i$

13. (1 point)

Let $f(x) = x^2 + 3x + 3$.

Evaluate the function for the following, write your answers in standard form $a + bi$

a) $f(4 + i)$

Answer: $30 + 11i$

b) $f(-i)$

Answer: $2 - 3i$

14. (1 point)

Find the absolute value of the complex number $-3 + 10i$

Answer: $\sqrt{109}$

15. (1 point)

Solve the equation: $-5(n - 3)^2 = 10$

Answer: $3 \pm i\sqrt{2}$

A bonus question: About the application of complex numbers in a real word problem.

- This question is considered as an **Extra credit** question.
- It costs **2** points.

ELECTRICITY. Circuit components such as resistors, inductors, and capacitors all oppose the flow of current. This opposition is called resistance for resistors and reactance for inductors and capacitors. A circuit's total opposition to current flow is impedance. All these quantities are measured in ohms (Ω). The impedance for a series circuit is the sum of the impedances for the individual components. Find the impedance of the circuit shown below:

