2/3 Mon

week 3

Theorem 12

Let $a, b, c, d, n \in \mathbb{Z}$ and $n \geq 2$.

If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ then

$$(a+c) \equiv (b+d) \pmod{n}$$

and $ac \equiv bd \pmod{n}$

Ex 13

$n = 4$

$0 \equiv 8 \pmod{4}$

$9 \equiv 21 \pmod{4}$

$8 \cdot 21 \equiv 0 \cdot 9 \equiv 0 \pmod{4}$

$(8+21) \equiv (0+9) \equiv 1 \pmod{4}$

$\boxed{9 \equiv 1 \pmod{4}}$

4)

4)

1

Ex 14 $n=23$

$$5^4 \equiv 5^2 \cdot 5^2 \equiv 25 \cdot 25 \pmod{23}$$
$$\equiv 2 \cdot 2 \pmod{23}$$
$$\equiv 4 \pmod{23}.$$

Another way

$$ q \quad n \quad + \; r$$
$$5^4 \equiv 625 \equiv (27)(23) + 4 \pmod{23}.$$
$$\equiv 0 + 4 \pmod{23}$$
$$\equiv 4 \pmod{23}$$

$23 \equiv 0 \pmod{23}$

MATH 4460.

mod 23

$$\begin{array}{r} 27 \\ 23\,\overline{)\,625} \\ -46 \\ \hline 165 \\ -161 \\ \hline 4 \end{array}$$

remainder

Def 15 Let p be an integer. Then p is **prime** if $p \geq 2$ and the only positive divisors of p are 1 and p.

Ex 16

positive divisors of 13 : 1, 13

So, 13 is prime

positive divisors of 12 : 1, 2, 3, 4, 6, 12

So, 12 is not prime.

12

Thm 17 (Euclid)

There are an infinite number of primes.

Some primes:

2, 3, 5, 7, 11, 13, 17,

19, 23, 29, ...

Thm 18 (Fundamental thm of arithmetic)

Let $x \in \mathbb{Z}$ with $x \geq 2$.

Then there exist distinct primes $p_1, p_2, \ldots, p_k$ and positive integers $e_1, e_2, \ldots, e_k$ such that

$$x = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$$

called the prime factorization of x

Furthermore, the above factorization is unique (up to reordering the above product).

Ex 19

$$42 = 2 \cdot 21 = 2^1 \cdot 3^1 \cdot 7^1$$

$$= 3^1 \cdot 2^1 \cdot 7^1$$

counts as same factorization

$$240 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$$

$$= 2^4 \cdot 3^1 \cdot 5^1$$

Thm 20

Let $x \in \mathbb{Z}$ with $x \geq 2$ and $x = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$ is the prime factorization of x. Then d is a divisor of x iff $d = p_1^{f_1} p_2^{f_2} \cdots p_k^{f_k}$ where $0 \leq f_i \leq e_i$ for all $i = 1, 2, \ldots, k$.

Ex 21:

$$x = 12 = 2^2 \cdot 3^1$$

divisors of x

$2^0 \cdot 3^0 = 1$

$2^0 \cdot 3^1 = 3$

$2^1 \cdot 3^0 = 2$

$2^1 \cdot 3^1 = 6$

$2^2 \cdot 3^0 = 4$

$2^2 \cdot 3^1 = 12$

Def 22: Let a and b be positive integers. We say that a and b are **relatively prime** if

$$\underbrace{\gcd}_{\text{greatest common divisor}}(a,b) = 1.$$

(That is, the only positive common divisor of a and b is 1.)

Ex 23

$a = 10$
$b = 21$

Positive divisors of 10: 1, 2, 5, 10
Positive divisors of 21: 1, 3, 7, 21
Common positive divisors: 1

$\gcd(10, 21) = 1$

10 & 21 are relatively prime

Ex 24

$a = 12$
$b = 18$

$\gcd(12, 18) = 6$

So, 12 and 18 are <u>not</u> relatively prime.

Def 25

A positive integer n is said to be *perfect* if n is equal to the sum of all its positive divisors (excluding the divisor n).

n).

Ex 26

$\boxed{n = 4}$

positive divisors not equal to 4

positive divisors: 1, 2, 4

$1 + 2 = 3 \neq 4$

4 is not perfect

$\boxed{n = 6}$

positive divisors not equal to 6

positive divisors: $\boxed{1, 2, 3}$, 6

$1 + 2 + 3 = 6$

6 is perfect

Ex 27 The next perfect number is 28 since

$$\underbrace{1+2+4+7+14}_{\text{positive divisors of 28, not equal to 28}} = 28$$

Ex 28 The first few perfect numbers are

6
28
496
8128
⋮