Tuesday
1/21

Review

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \cos(x)\,dx = \sin(x) + C$$

$$\int \sin(x)\,dx = -\cos(x) + C$$

$$\int a^x dx = \frac{1}{\ln(a)} \cdot a^x + C$$

$$\int e^x dx = e^x + C$$

<u>FTC</u>: If F is an antiderivative of f, that is $F' = f$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Ex:

$$\int_0^1 \left[2x^2 + \sin(x)\right] dx$$

$$= \left(2\frac{x^3}{3} - \cos(x)\right)\Bigg|_0^1$$

$$= \left(\frac{2}{3}(1)^3 - \cos(1)\right)$$

$$- \left(\frac{2}{3}(0)^2 - \cos(0)\right)$$

$$= \frac{2}{3} - \cos(1) + 1$$

$$= \frac{5}{3} - \cos(1) \approx 1.12636\ldots$$

Ex: $\int e^{2x}\,dx = \int \frac{1}{2} e^{u}\,du = \frac{1}{2} e^{u} + C$

$= \frac{1}{2} e^{2x} + C$

$u = 2x$
$du = 2\,dx$
$\frac{1}{2}du = dx$

If $F' = f$, then

$$\int f(cx)\,dx = \frac{1}{c} F(cx) + C$$

Ex: $\int \cos(10x)\,dx$

$= \frac{1}{10}\sin(10x) + C$

Ex: $\int \sin(\pi x)\,dx$

$$= -\frac{1}{\pi}\cos(\pi x) + C$$

$$\int g(h(x))\,h'(x)\,dx$$

$$\underset{\uparrow}{=} \int g(u)\,du$$

$u = h(x),\ du = h'(x)\,dx$

Ex: $\int x\sqrt{x^2+5}\,dx$

$$\underset{\uparrow}{=} \int \tfrac{1}{2}\sqrt{u}\,du$$

$u = x^2+5$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\int u^{1/2}\,du$$

$$= \frac{1}{2}\,\frac{u^{1/2+1}}{(1/2+1)} + C$$

C

$$\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C$$

$$=\frac{1}{3}\left(x^2+5\right)^{3/2}+C$$

Check:

$$\frac{d}{dx}\left[\frac{1}{3}\left(x^2+5\right)^{3/2}\right]$$

$$\frac{1}{3}\cdot\frac{3}{2}\left(x^2+5\right)^{1/2}\cdot(2x)$$

$$=x\sqrt{x^2+5}$$

7.2 – Integration by Parts

$$\left(f(x)\, g(x) \right)' = f'(x)\, g(x) + f(x)\, g'(x)$$

Integrate

$$f(x)\, g(x) = \int f'(x)\, g(x)\, dx + \int f(x)\, g'(x)\, dx$$

$$\int \underbrace{f(x)}_{u}\,\underbrace{g'(x)\,dx}_{dv} = \underbrace{f(x)}_{u}\,\underbrace{g(x)}_{v} - \int \underbrace{g(x)}_{v}\,\underbrace{f'(x)\,dx}_{du}$$

$$\int u\,dv = uv - \int v\,du$$

dx