Thursday
1/23

$k = K$

7.3 - Trigonometric Integrals

Strategy for $\int \sin^m(x)\cos^n(x)\,dx$

① m is odd case

So, $m = 2k+1$.

factor out one $\sin(x)$

$$\int \sin^{2k+1}(x)\cos^n(x)\,dx = \int \sin^{2k}(x)\cos^n(x)\sin(x)\,dx$$

$$= \int (\sin^2(x))^k \cos^n(x)\sin(x)\,dx = \int (1-\cos^2(x))^k \cos^n(x)\sin(x)\,dx$$

let $u = \cos(x)$
$du = -\sin(x)dx$
$-du = \sin(x)dx$

$= -\int ($

ometric Integrals

r $\int \sin^m(x)\cos^n(x)\,dx$

ase

+1.

factor out one sin(x)

$(x)dx = \int \sin^{2k}(x)\cos^n(x)\sin(x)\,dx$

$x)\sin(x)dx = \int (1-\cos^2(x))^k \cos^n(x)\sin(x)\,dx$

let $u=\cos(x)$
$du = -\sin(x)dx$
$-du = \sin(x)dx$

$-\int (1-u^2)^k u^n\,du$

Now integrate.

(2) n is odd case

Do the same idea as in step (1) but instead factor out a cosine. Turn the remaining cosines into sines using $\cos^2(X) = 1-\sin^2(X)$. Then do the sub $u=\sin(X)$.

③ If both m and n are even, then use the formulas

$$\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$$

You may need to use the formulas more than once.

7.3 – Trig

Strategy

① m is odd

So, m = 2

$$\int \sin^{2k+1}(x)\cos$$

$$= \int (\sin^2(x))^k \cos$$

Ex:

$$\int \cos^3(x)\,dx = \int \underbrace{\cos^2(x)}_{\text{turn into sines}}\ \underbrace{\cos(x)\,dx}_{\text{keep for du}}$$

$$= \int \left(1-\sin^2(x)\right)\cos(x)\,dx$$

$$\boxed{\begin{gathered} u = \sin(x) \\ du = \cos(x)\,dx \end{gathered}}$$

$$= \int (1-u^2)\,du = u - \frac{u^3}{3} + C$$

$$= \boxed{\sin(x) - \frac{1}{3}\sin^3(x) + C}$$

Ex:

$$\int \sin^5(x)\cos^2(x)\,dx$$

$$= \int \underbrace{\sin^4(x)\cos^2(x)}_{\text{turn into cosines}}\ \underbrace{\sin(x)\,dx}_{\text{save for } du}$$

$$= \int \left(\sin^2(x)\right)^2 \cos^2(x)\,\sin(x)\,dx$$

$$= \int (1-\cos^2(x))^2 \cos^2(x) \sin(x)\,dx$$

$$= -\int (1-u^2)^2 u^2\,du$$

$u = \cos(x)$
$du = -\sin(x)dx$
$-du = \sin(x)dx$

$$= -\int (1-2u^2+u^4)u^2\,du$$

$$\int (-u^6+2u^4-u^2)\,du$$

$$= -\frac{u^7}{7} + 2\frac{u^5}{5} - \frac{u^3}{3} + C$$

$$= -\frac{1}{7}\cos^7(x) + \frac{2}{5}\cos^5(x) - \frac{1}{3}\cos^3(x) + C$$

$$\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$$

Ex:

$$\int \sin^4(x)\,dx = \int \left(\sin^2(x)\right)^2 dx$$

$$= \int \left(\frac{1}{2} - \frac{1}{2}\cos(2x)\right)^2 dx = \int \left(\frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{1}{4}\cos^2(2x)\right) dx$$

$$= \int \left(\frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{1}{4}\left[\frac{1}{2} + \frac{1}{2}\cos(4x)\right]\right) dx$$

$$= \int \left(\frac{3}{8} - \frac{1}{2}\cos(2x) + \frac{1}{8}\cos(4x) \right) dx$$

$$= \frac{3}{8}x - \frac{1}{2} \cdot \frac{1}{2}\sin(2x) + \frac{1}{8} \cdot \frac{1}{4}\sin(4x) + C$$

$$= \frac{3}{8}x - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C$$

Last time: $\int \tan(x)\,dx = \ln|\sec(x)| + C$

So,

$\int \sec(x)\,dx$

$= \ln|\tan(x)+\sec(x)| + C$

Ex: $\int \sec(x)\,dx = \int \sec(x) \cdot \underbrace{\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}}_{1}\,dx$

$= \int \frac{\sec^2(x)+\sec(x)\tan(x)}{\tan(x)+\sec(x)}\,dx = \int \frac{du}{u} = \ln|u| + C$

$u = \tan(x)+\sec(x)$
$du = (\sec^2(x)+\sec(x)\tan(x))\,dx$

$= \boxed{\ln|\tan(x)+\sec(x)| + C}$