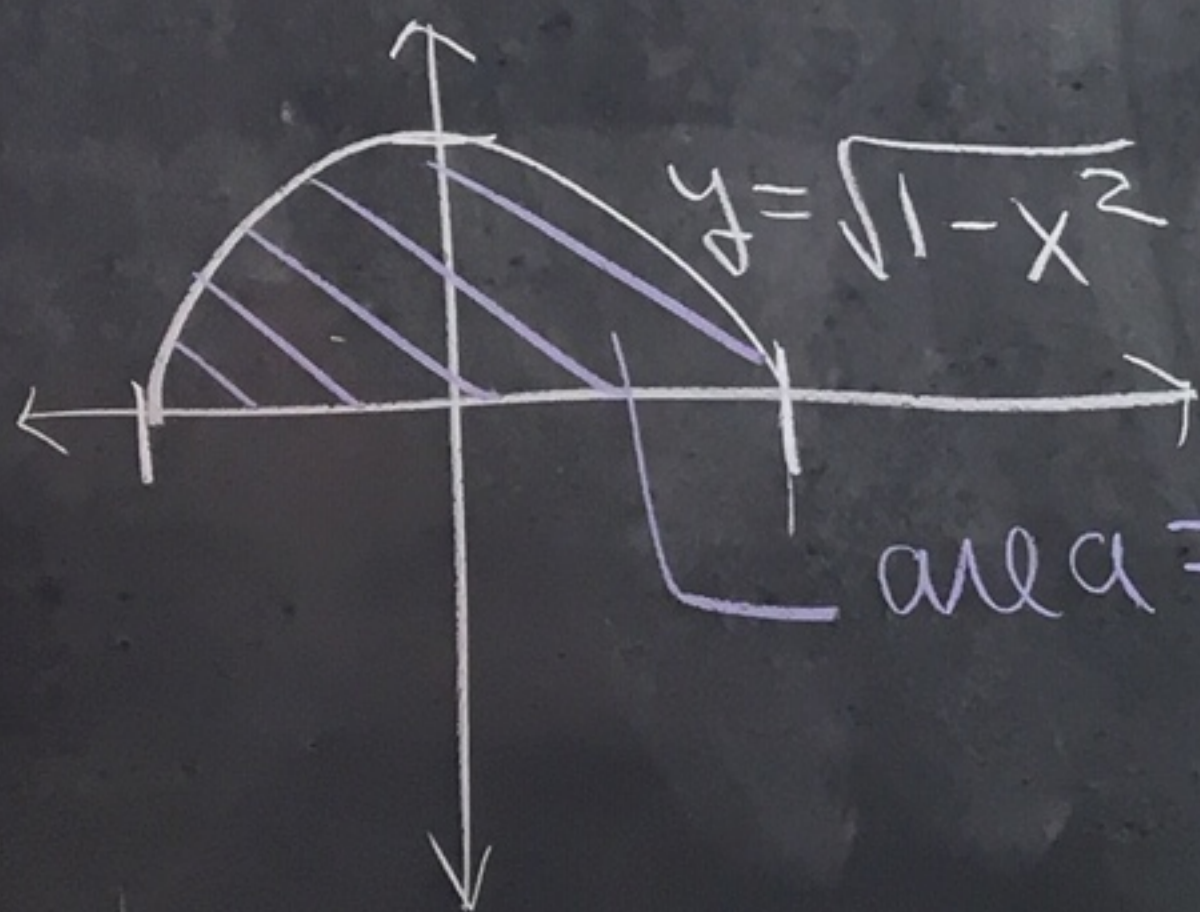


Monday
1/27

$$y^2 + x^2 = 1$$
$$y = \pm \sqrt{1-x^2}$$



$$\text{area} = \frac{\pi(1)^2}{2} = \frac{\pi}{2}$$

7.4 - Trig. sub.

Ex: Evaluate $\int_{-1}^1 \sqrt{1-x^2} dx$

Let $x = \sin(\theta)$
 $dx = \cos(\theta) d\theta$

$$\int_{-1}^1 \sqrt{1-x^2} dx = \int_{-1}^1 \sqrt{1-\sin^2(\theta)} \cos(\theta) d\theta$$

(Note: The diagram shows the limits of integration for the theta integral as 0 to pi, with arrows pointing from the x-axis limits to the theta limits.)

Com
revo
 \int
 $\frac{1}{u}$
 $\frac{du}{u}$

$$\int_{-1}^1 \sqrt{1-x^2} dx$$

θ .

$$\sqrt{1-\sin^2(\theta)} \cos(\theta) d\theta$$

θ

What are ?'s ?

Need to first pick a domain for sine where its 1-1

Commentary: This is just a reverse u-sub.

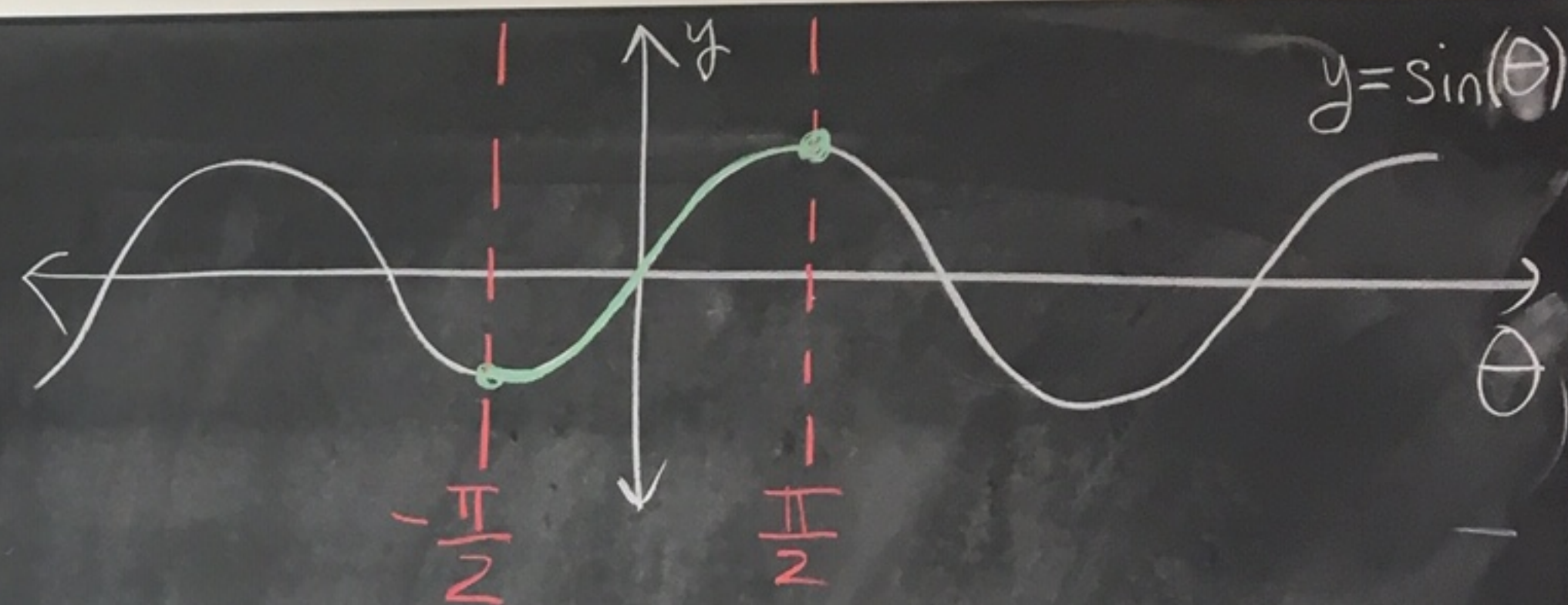
$$\int \sqrt{1-\sin^2(\theta)} \cos(\theta) d\theta$$

$$= \int \sqrt{1-u^2} du$$

$$u = \sin(\theta)$$

$$du = \cos(\theta) d\theta$$

what we started with



We restrict $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

Use $x = \sin(\theta)$.

When $x=1$, $\sin(\theta)=1$. So, $\theta = \frac{\pi}{2}$.

When $x=-1$, $\sin(\theta)=-1$. So, $\theta = -\frac{\pi}{2}$.

$$\int_{-1}^1 \sqrt{1-x^2} dx = \int_{-\pi/2}^{\pi/2} \sqrt{1-\sin^2(\theta)} \cos(\theta) d\theta$$

$$\cos(\theta) d\theta$$

$$\int_{-\pi/2}^{\pi/2} \sqrt{\cos^2(\theta)} \cos(\theta) d\theta$$

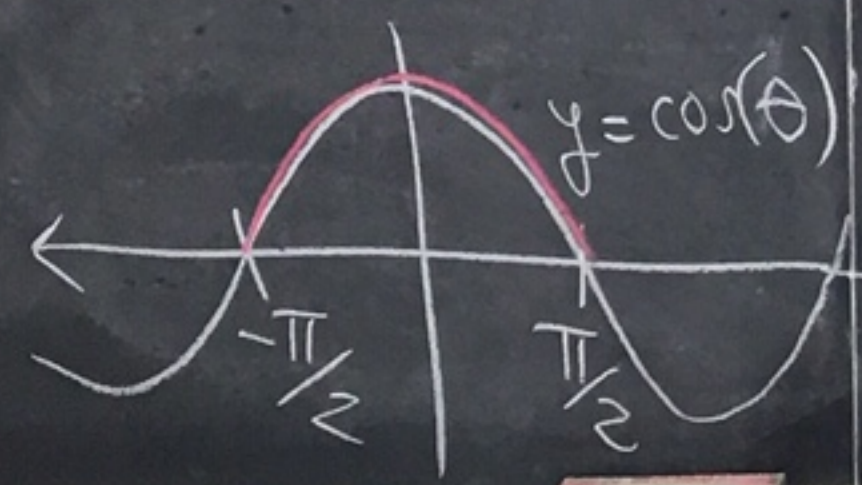
remove the $\sqrt{1-\sin^2(\theta) = \cos^2(\theta)}$

$$\int_{-\pi/2}^{\pi/2} \cos(\theta) \cos(\theta) d\theta$$

$$\sqrt{(-1)^2} = 1$$

$$\sqrt{(-1)^2} \neq -1$$

$x = \sin(\theta)$
 $dx = \cos(\theta) d\theta$
 $x = 1 \rightarrow \theta = \pi/2$
 $x = -1 \rightarrow \theta = -\pi/2$



$\sqrt{x^2} = |x|$
 When $-\pi/2 \leq \theta \leq \pi/2$, $\cos(\theta) \geq 0$
 So, $\sqrt{\cos^2(\theta)} = |\cos(\theta)| = \cos(\theta)$

$$\int_{-\pi/2}^{\pi/2} \cos^2(\theta) d\theta = \left[\frac{\theta}{2} \right]_{-\pi/2}^{\pi/2}$$

$$\int_{-\pi/2}^{\pi/2} \cos^2(\theta) d\theta$$

$$\int_{-\pi/2}^{\pi/2} \cos(\theta) d\theta$$

$$\geq 0 \\ = \cos(\theta)$$

$$\int_{-\pi/2}^{\pi/2} \cos^2(\theta) d\theta = \int_{-\pi/2}^{\pi/2} \left[\frac{1}{2} + \frac{1}{2} \cos(2\theta) \right] d\theta$$

$$= \left[\frac{1}{2} \theta + \frac{1}{2} \cdot \frac{1}{2} \sin(2\theta) \right] \Big|_{-\pi/2}^{\pi/2}$$

$$= \left[\frac{1}{2} \cdot \frac{\pi}{2} + \frac{1}{4} \underbrace{\sin\left(2 \cdot \frac{\pi}{2}\right)}_{\sin(\pi) = 0} \right] - \left[\frac{1}{2} \left(-\frac{\pi}{2}\right) + \frac{1}{4} \underbrace{\sin\left(2 \left(-\frac{\pi}{2}\right)\right)}_{\sin(-\pi) = 0} \right] = \frac{\pi}{4} + \frac{\pi}{4} = \left(\frac{\pi}{2} \right)$$

Table for trig subs

to get rid of $\sqrt{\quad}$

Expression	substitution	identity used after substitution
$\sqrt{a^2 - x^2}$	$x = a \sin(\theta)$ $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	$1 - \sin^2(\theta)$ $= \cos^2(\theta)$
$\sqrt{a^2 + x^2}$	$x = a \tan(\theta)$ $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \tan^2(\theta)$ $= \sec^2(\theta)$
$\sqrt{x^2 - a^2}$	$x = a \sec(\theta)$ $0 \leq \theta < \frac{\pi}{2}$ or $\pi \leq \theta < \frac{3\pi}{2}$	$\sec^2(\theta) - 1$ $= \tan^2(\theta)$

7.2

$$[\ln(x)]^2$$

$$\begin{aligned} u &= \ln(x) \\ dv &= x^2 dx \\ du &= \frac{1}{x} dx \\ v &= \frac{x^3}{3} \end{aligned}$$

$$(26) \int x^2 \ln^2(x) dx$$

$$\frac{x^3}{3} \ln^2(x) - \frac{2}{3} \int x^2 \ln(x) dx$$

$$\begin{aligned} u &= [\ln(x)]^2 & du &= 2[\ln(x)] \cdot \frac{1}{x} \\ dv &= x^2 dx & v &= \frac{x^3}{3} \end{aligned}$$

$$\frac{x^3}{3} \ln^2(x) - \frac{2}{3} \left[\frac{x^3}{3} \ln(x) - \int \frac{1}{3} x^2 dx \right]$$

$$\begin{aligned} &= \frac{x^3}{3} \ln^2(x) - \frac{2}{9} x^3 \ln(x) \\ &\quad + \frac{2}{27} x^3 + C \end{aligned}$$

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SHELTER
IN PLACE
POWER
OUTAGE
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