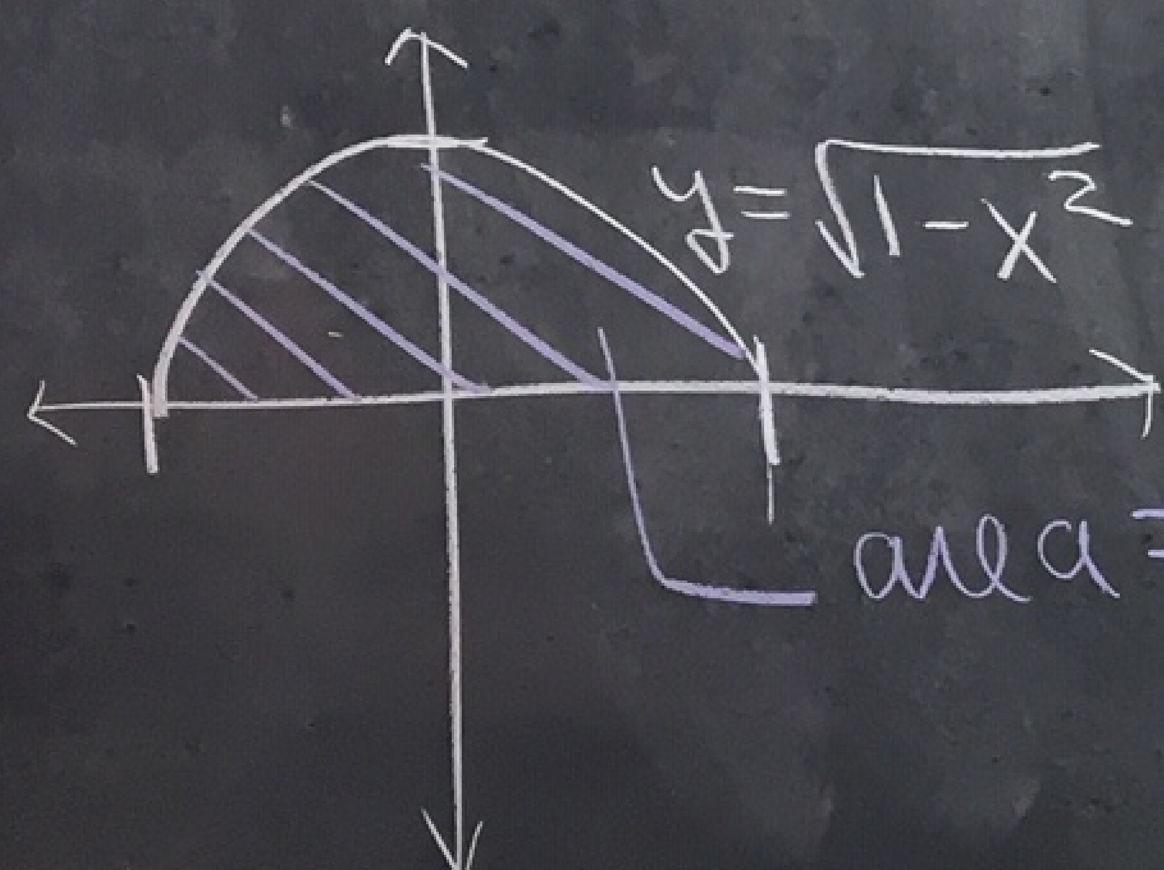
Monday
1/27

$y^2+x^2=1$

$y=\pm\sqrt{1-x^2}$



area $= \frac{\pi(1)^2}{2} = \frac{\pi}{2}$

7.4 – Trig. sub.

Ex: Evaluate $\int_{-1}^{1}\sqrt{1-x^2}\,dx$

Let $x=\sin(\theta)$

$dx=\cos(\theta)\,d\theta$.

$$\int_{-1}^{1}\sqrt{1-x^2}\,dx=\int_{?}^{?}\sqrt{1-\sin^2(\theta)}\,\cos(\theta)\,d\theta$$

Com

reve

$u=$

$du=$

$$\int_{-1}^{1}\sqrt{1-x^2}\,dx$$

θ.

$\sqrt{1-\sin^2(\theta)}\;\cos(\theta)\,d\theta$

?

What are ?'s ?

Need to first pick a domain for sine where its 1-1

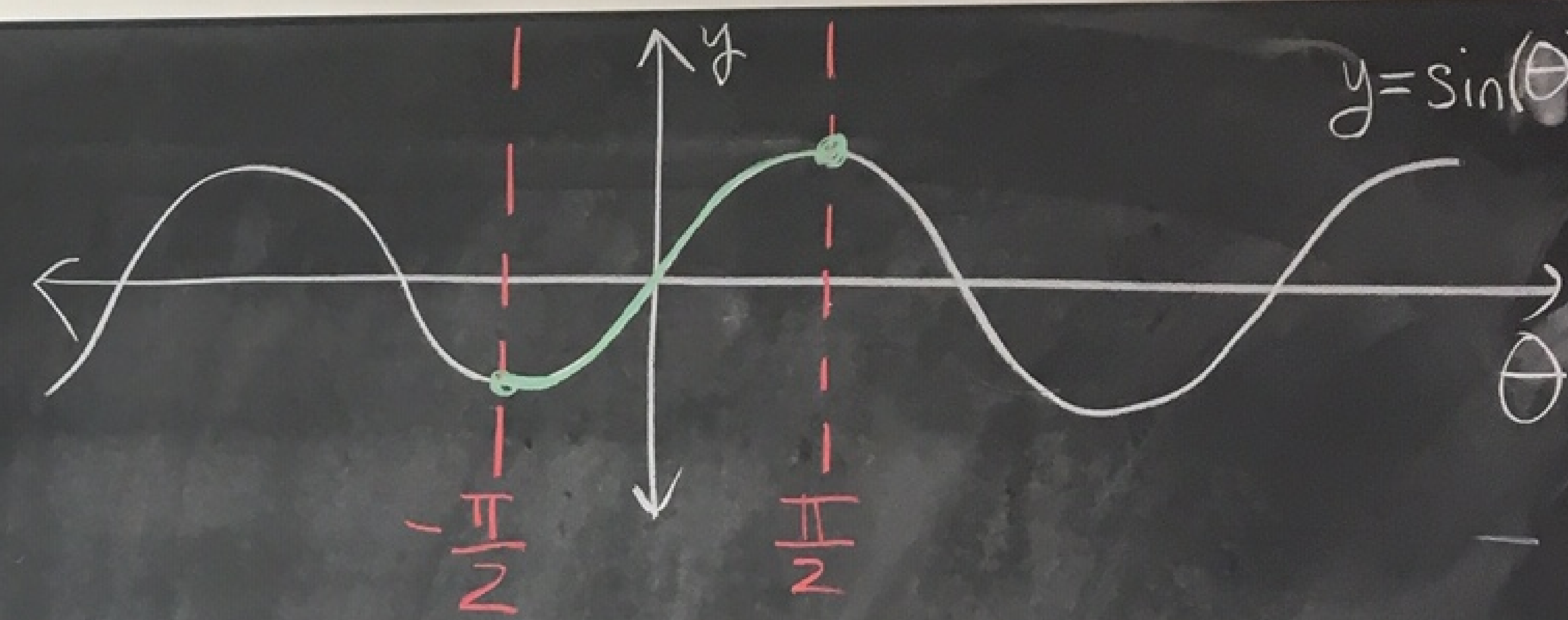
Commentary: This is just a reverse u-sub.

$$\int\sqrt{1-\sin^2(\theta)}\,\cos(\theta)\,d\theta = \int\sqrt{1-u^2}\,du$$

$u=\sin(\theta)$

$du=\cos(\theta)\,d\theta$

What we started with



We restrict $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$.

Use $x=\sin(\theta)$.

When $x=1$, $\sin(\theta)=1$. So, $\theta=\frac{\pi}{2}$.

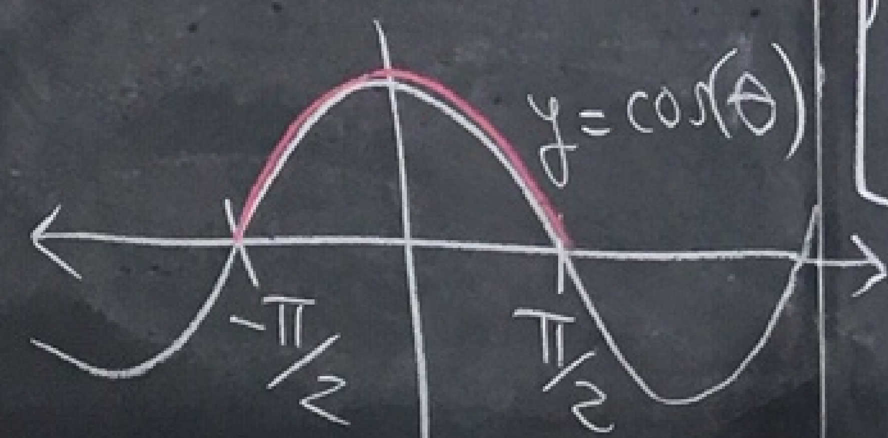
When $x=-1$, $\sin(\theta)=-1$. So, $\theta=\frac{-\pi}{2}$.

$$\int_{-1}^{1}\sqrt{1-x^2}\,dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sqrt{1-\sin^2(\theta)}\,\cos(\theta)\,d\theta = \int_{-\pi/2}^{\pi/2}\sqrt{\cos^2(\theta)}\,\cos(\theta)\,d\theta = \int_{-\pi/2}^{\pi/2}\cos(\theta)\cos(\theta)\,d\theta$$

$x = \sin(\theta)$
$dx = \cos(\theta)\,d\theta$
$x = 1 \longrightarrow \theta = \frac{\pi}{2}$
$x = -1 \longrightarrow \theta = -\frac{\pi}{2}$

remove the $\sqrt{\ }$
$1-\sin^2(\theta) = \cos^2(\theta)$

$\sqrt{(-1)^2} = 1$
$\sqrt{(-1)^2} \neq -1$



$\sqrt{x^2} = |x|$
When $-\pi/2 \le \theta \le \pi/2$, $\cos(\theta) \ge 0$
So $\sqrt{\cos^2(\theta)} = |\cos(\theta)| = \cos(\theta)$

$= \left[\frac{1}{2}\cdot\frac{\pi}{2}\right.$

$$\ldots^{2}(\theta)\ \cos(\theta)\,d\theta =$$

$$\ldots s(\theta)\cos(\theta)\,d\theta =$$

$$\geq 0$$

$$= \cos(\theta)$$

$$\int_{-\pi/2}^{\pi/2} \cos^2(\theta)\,d\theta = \int_{-\pi/2}^{\pi/2} \left[\frac{1}{2} + \frac{1}{2}\cos(2\theta)\right] d\theta$$

$$= \left[\frac{1}{2}\theta + \frac{1}{2}\cdot\frac{1}{2}\sin(2\theta)\right]\Bigg|_{-\pi/2}^{\pi/2}$$

$$= \left[\frac{1}{2}\cdot\frac{\pi}{2} + \frac{1}{4}\underbrace{\sin\left(2\cdot\frac{\pi}{2}\right)}_{\sin(\pi)=0}\right] - \left[\frac{1}{2}\left(-\frac{\pi}{2}\right) + \frac{1}{4}\underbrace{\sin\left(2\left(-\frac{\pi}{2}\right)\right)}_{\sin(-\pi)=0}\right] = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$

Table for trig subs

to get rid of $\sqrt{\ }$

Expression	Substitution	identity used after substitution
$\sqrt{a^2-x^2}$	$x = a\sin(\theta)$ $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$	$1-\sin^2(\theta)$ $=\cos^2(\theta)$
$\sqrt{a^2+x^2}$	$x = a\tan(\theta)$ $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1+\tan^2(\theta)$ $=\sec^2(\theta)$
$\sqrt{x^2-a^2}$	$x = a\sec(\theta)$ $0 \le \theta < \frac{\pi}{2}$ or $\pi \le \theta < \frac{3\pi}{2}$	$\sec^2(\theta)-1$ $=\tan^2(\theta)$

7.2

$$\underbrace{[\ln(x)]^2}$$

26 $\int x^2 \ln^2(x)\,dx$

$$u = \ln(x) \qquad dv = x^2\,dx \qquad du = \frac{1}{x}\,dx \qquad v = x^3/3$$

$$= \frac{x^3}{3}\ln^2(x) - \frac{2}{3}\int x^2 \ln(x)\,dx$$

$$u = [\ln(x)]^2 \qquad du = 2[\ln(x)]^1 \cdot \frac{1}{x}$$

$$dv = x^2\,dx \qquad v = \frac{x^3}{3}$$

$$\frac{x^3}{3}\ln^2(x) - \frac{2}{3}\left[\frac{x^3}{3}\ln(x) - \int \frac{1}{3}x^2\,dx\right]$$

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$$= \frac{x^3}{3}\ln^2(x) - \frac{2}{9}x^3\ln(x) + \frac{2}{27}x^3 + C$$