Mon.
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(7.5 continued...)

#21 Evaluate $\int \frac{6x^2}{x^4-5x^2+4}\,dx$

Step 1: bottom degree is 4
top degree is 2
So no need to divide
bottom into top.

$\quad dx$

Step 2: Factor the denominator

To make it easier let $u = x^2$. Then

$$x^4 - 5x^2 + 4 = u^2 - 5u + 4$$
$$= (u-4)(u-1)$$
$$= (x^2-4)(x^2-1)$$

$$= \underbrace{(x+2)(x-2)(x+1)(x-1)}_{\text{No repeated factors}}$$

Since we have no repeated factors we can do the following:

$$\frac{6x^2}{x^4 - 5x^2 + 4} = \frac{6x^2}{(x+2)(x-2)(x+1)(x-1)}$$

$$= \frac{A}{x+2} + \frac{B}{x-2} + \frac{C}{x+1} + \frac{D}{x-1}$$

Multiply both sides of the previous equation by $(x+2)(x-2)(x+1)(x-1)$ to get

$$(*) \quad \boxed{\begin{aligned} 6x^2 = {} & A(x-2)(x+1)(x-1) + B(x+2)(x+1)(x-1) \\ & + C(x+2)(x-2)(x-1) + D(x+2)(x-2)(x+1) \end{aligned}}$$

Find A, by plugging in $x=-2$ into $(*)$

$$6(-2)^2 = A(-2-2)(-2+1)(-2-1) + B(0) + C(0) + D(0)$$

$$24 = A(-12) \rightarrow \boxed{A=-2}$$

Find B, by plugging in $x=2$ into $(*)$

$$6(2)^2 = A(0) + B(2+2)(2+1)(2-1) + C(0) + D(0)$$

$$24 = 12B \rightarrow \boxed{B=2}$$

Find C, by plugging in $x=-1$ into $(*)$

$$6(-1)^2 = A(0) + B(0) + C(-1+2)(-1-2)(-1-1) + D(0)$$

$$6 = C(6) \rightarrow \boxed{C=1}$$

Find D, by plugging $x=1$ into $(*)$

$$6(1)^2 = A(0) + B(0) + C(0) + D(1+2)(1-2)(1+1)$$

$$6 = D(-6) \rightarrow \boxed{D=-1}$$

So, $\displaystyle\int \frac{6x^2}{x^4-5x^2+4}\,dx = \int\left(\frac{A}{x+2}+\frac{B}{x-2}+\frac{C}{x+1}+\frac{D}{x-1}\right)dx$

$\displaystyle = \int\left(\frac{-2}{x+2}+\frac{2}{x-2}+\frac{1}{x+1}+\frac{-1}{x-1}\right)dx$

$\boxed{\displaystyle\int\frac{dx}{ax+b}=\frac{1}{a}\ln|ax+b|+C}$

$= \boxed{-2\ln|x+2|+2\ln|x-2|+\ln|x+1|-\ln|x-1|+C}$

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$= \ln\left(|x+2|^{-2}\right)+\ln\left(|x-2|^{2}\right)+\ln|x+1|+\ln\left(|x-1|^{-1}\right)$

$\boxed{\ln(ab)=\ln(a)+\ln(b)}$

$\displaystyle \ln\left(\frac{|x-2|^2\,|x+1|}{|x+2|^2\,|x-1|}\right)$

7.8 - Improper Integrals

Ex: Can we make sense of $\int_1^\infty \frac{1}{x^2}\,dx$?

Idea: Let $t > 1$.

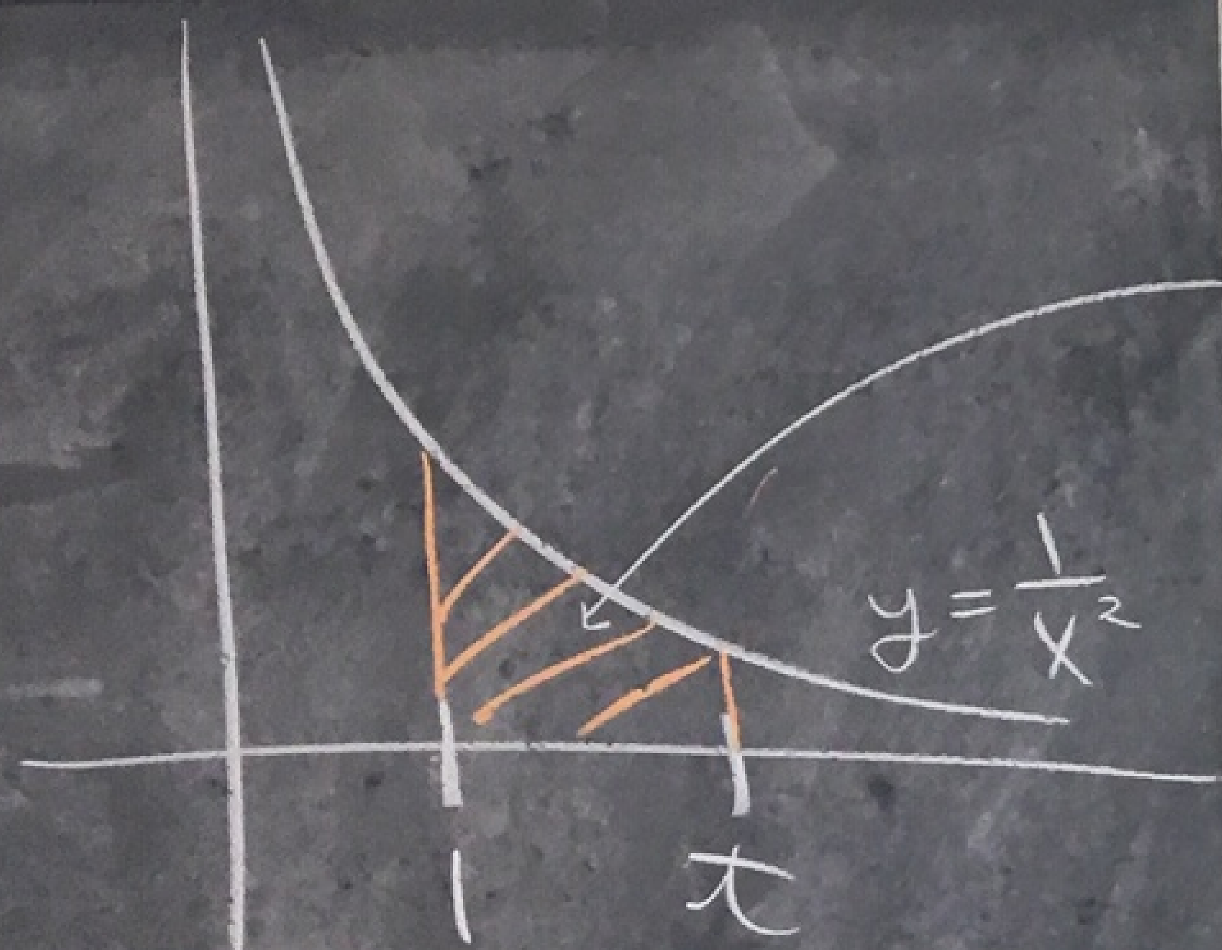
area $= \int_1^t \frac{1}{x^2}\,dx$

Calculate $\int_1^t \frac{1}{x^2}\,dx$ and you get a number based on t.

Define

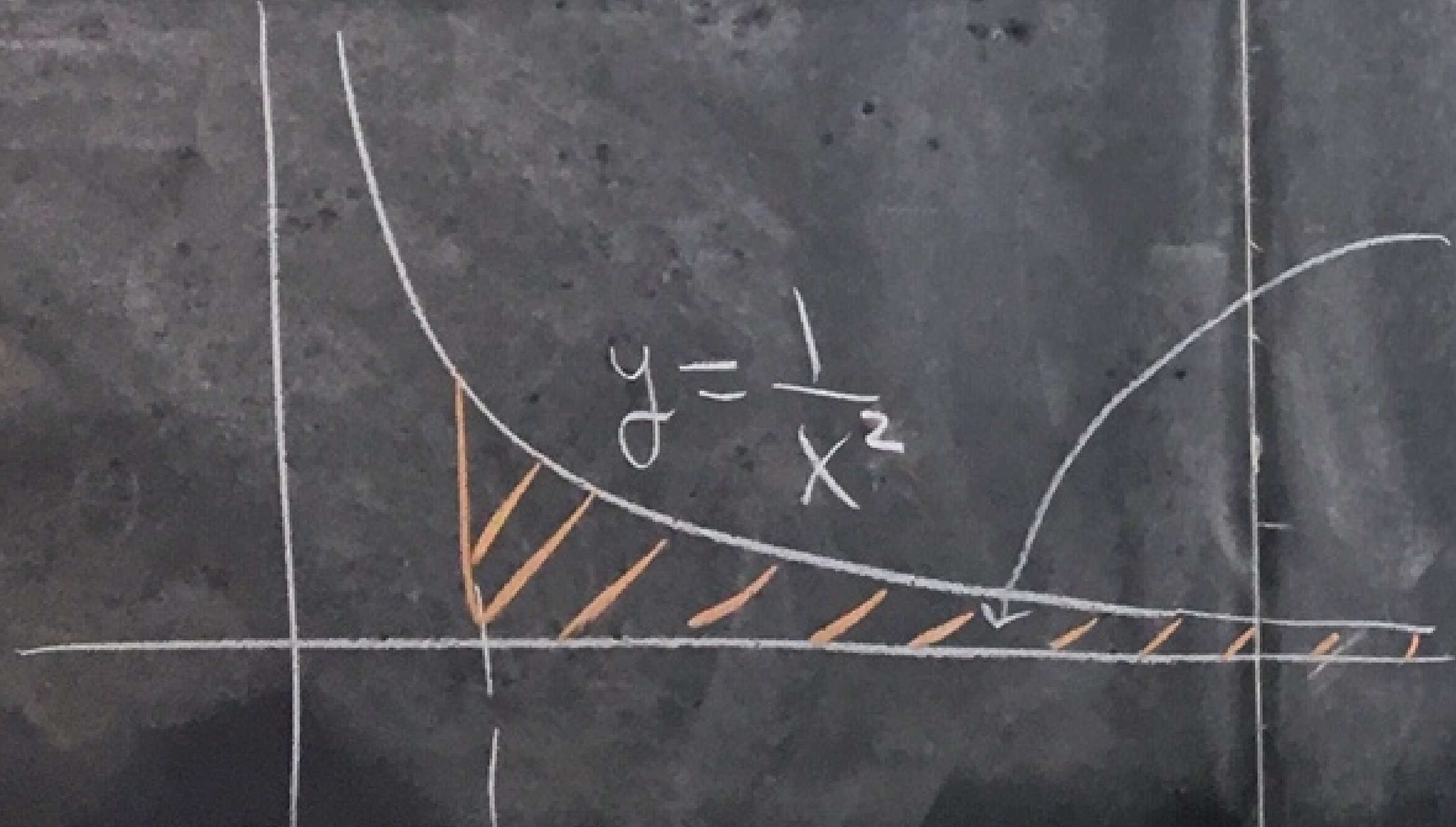
$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{t \to \infty} \left(\int_1^t \frac{1}{x^2}\,dx \right)$$

if the limit exists.



$$\text{area} = \int_1^t \frac{1}{x^2}\,dx = \int_1^t x^{-2}\,dx = \left(\frac{x^{-1}}{-1}\right)_1^t$$

$$= \left(-\frac{1}{t} - \left(-\frac{1}{1}\right)\right) = 1 - \frac{1}{t}$$



$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{x^2}\,dx = \lim_{t\to\infty}\left(1 - \frac{1}{t}\right)$$

$$= 1 - 0 = \boxed{1}$$