Tuesday
2/4

(7.8 continued...)

Def: (Improper integrals on infinite intervals)

① If $\int_a^t f(x)\,dx$ exists for all $a \leq t$, then we define

$$\int_a^\infty f(x)\,dx = \lim_{t \to \infty} \int_a^t f(x)\,dx$$

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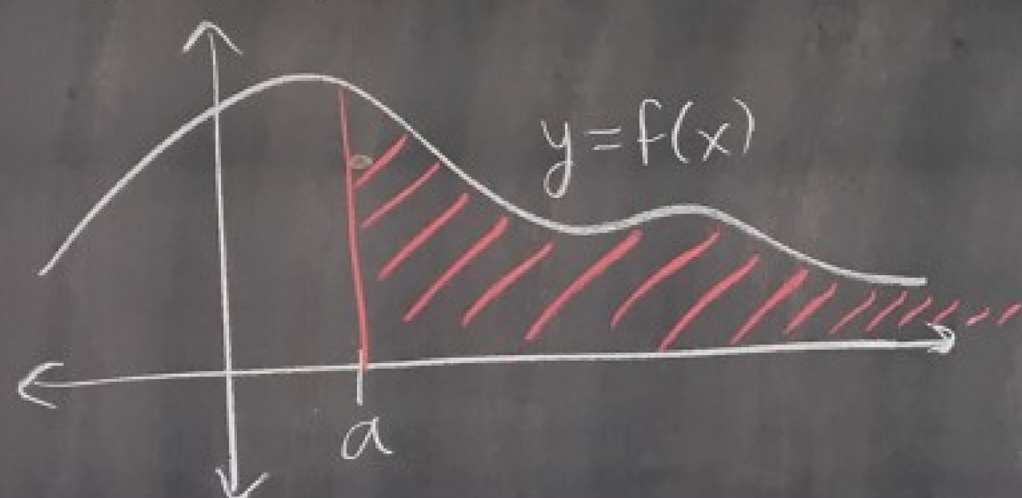
dx

if this limit exists.

That is, to compute



we compute the following as $t \to \infty$

If the limit exists we say that $\int_a^\infty f(x)\,dx$ is <u>convergent</u>. Otherwise we say that $\int_a^\infty f(x)\,dx$ is <u>divergent</u>

(2) If $\int_t^a f(x)\,dx$

exists for all $t \leq a$
then we define

$$\int_{-\infty}^{a} f(x)\,dx = \lim_{t \to -\infty} \int_t^a f(x)\,dx$$

if the limit exists.

That is, to compute

we compute the following as $t \to -\infty$

If the limit exists we say that $\int_{-\infty}^{a} f(x)\,dx$ is <u>convergent</u>,

otherwise we say that $\int_{-\infty}^{a} f(x)\,dx$ is <u>divergent</u>

③ If both $\int_{-\infty}^{a} f(x)\,dx$ and $\int_{a}^{\infty} f(x)\,dx$

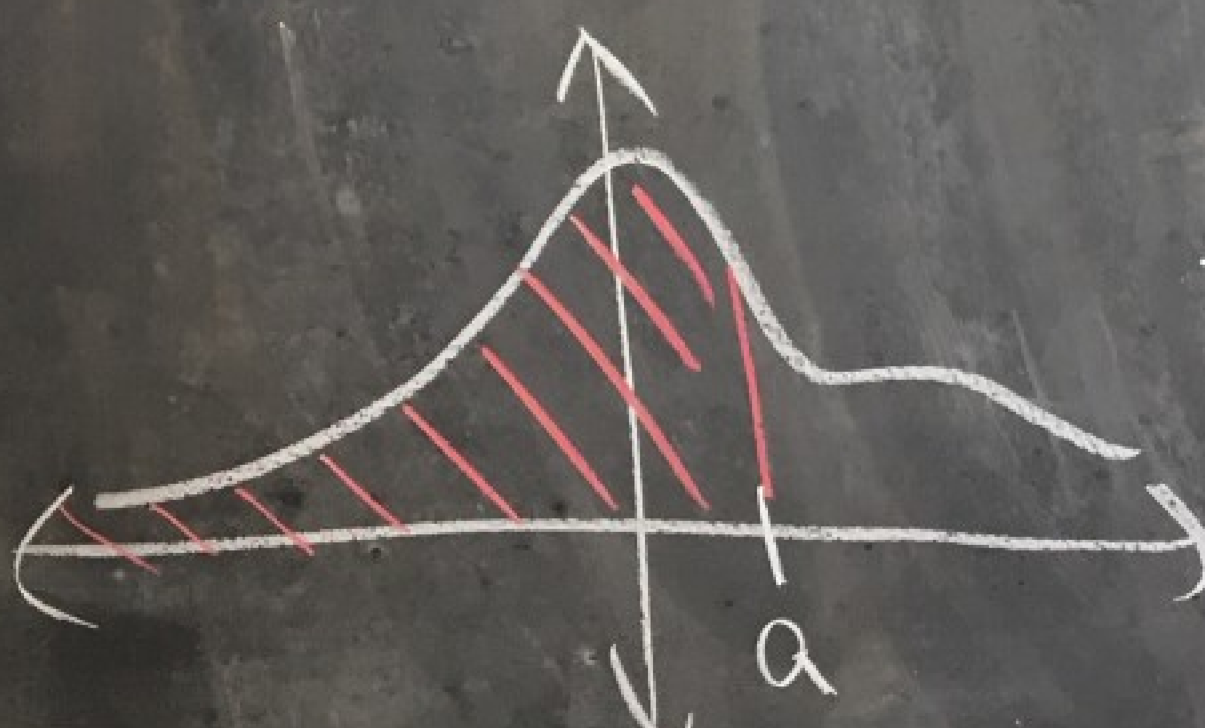
exists for some a, then we define

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{a} f(x)\,dx + \int_{a}^{\infty} f(x)\,dx$$

(any a can be chosen)

To compute

We add these



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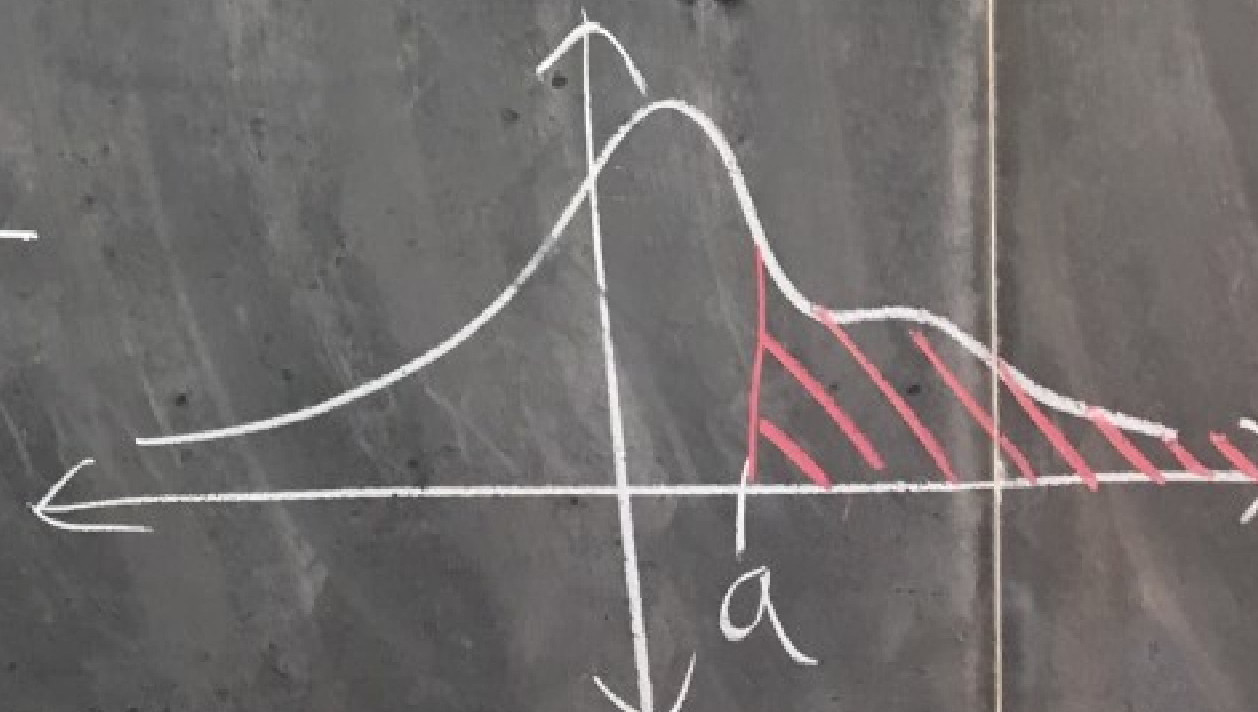
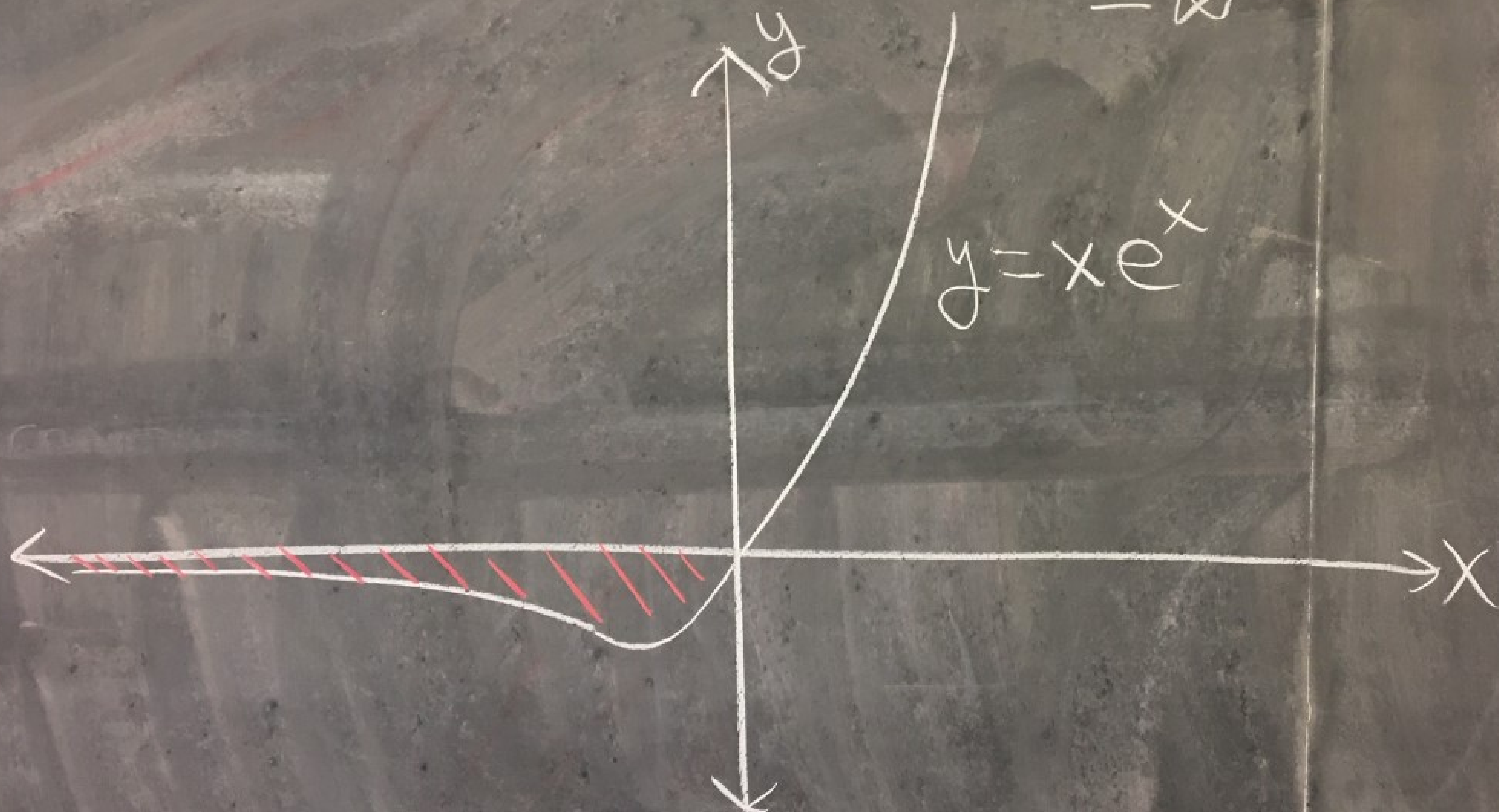
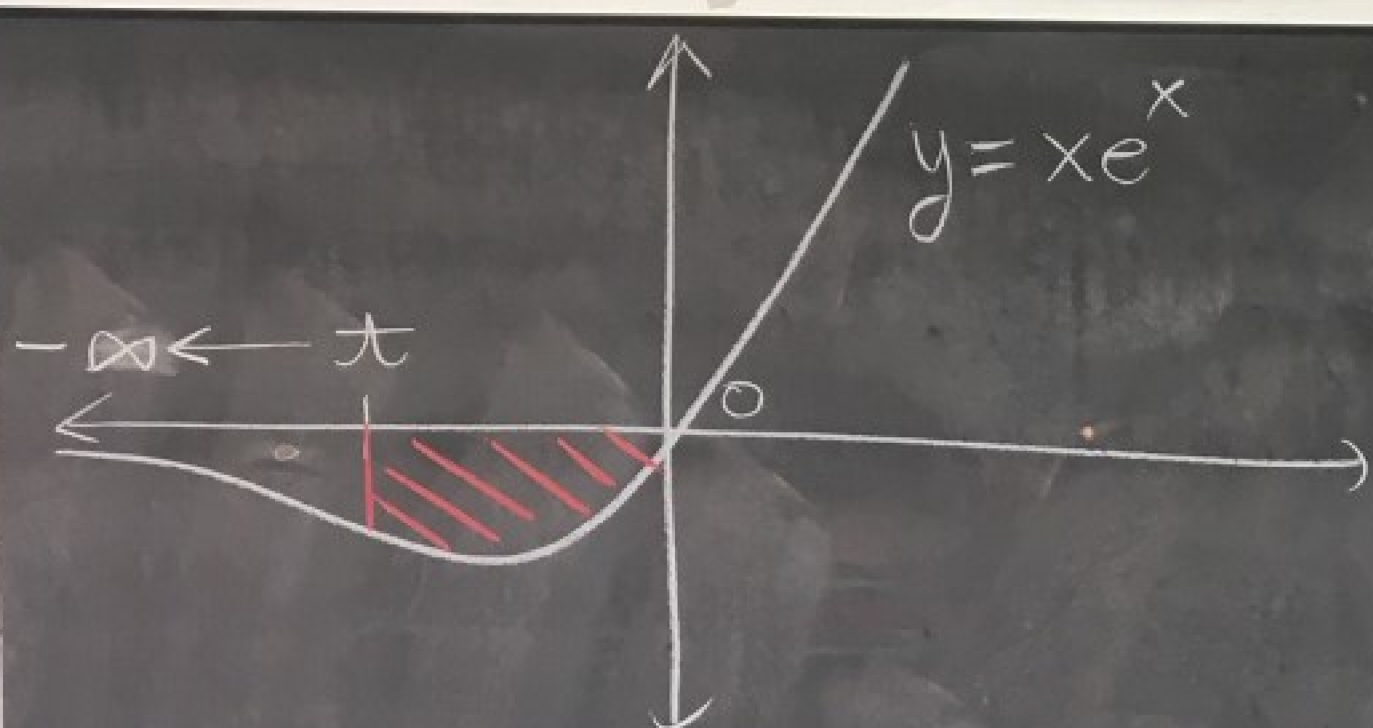


Ex: Evaluate $\int_{-\infty}^{0} xe^x \, dx$





$$\int_t^0 xe^x\,dx = xe^x\Big|_t^0 - \int_t^0 e^x\,dx$$

$u = x \qquad du = dx$

$dv = e^x\,dx \qquad v = e^x$

$$= (0 - te^t) - (e^x)\Big|_t^0$$

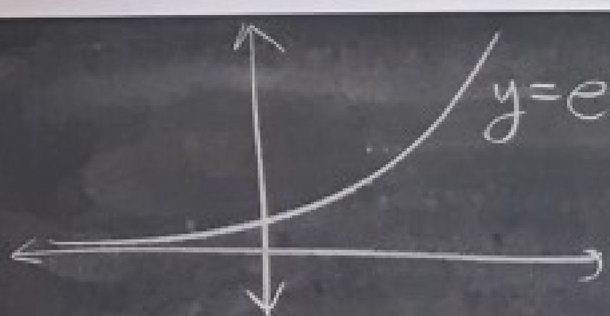
$$= -te^t - (e^0 - e^t)$$

$$= \boxed{-te^t - 1 + e^t}$$

So, $\displaystyle\int_{-\infty}^0 xe^x\,dx = \lim_{t\to-\infty}\int_t^0 xe^x\,dx$

$$= \lim_{t\to-\infty}\left(-te^t - 1 + e^t\right)$$

if this limit exists.



$$\lim_{t\to-\infty} e^t = 0$$

What about this one:

$$\lim_{t\to-\infty}(te^t) = \lim_{t\to-\infty}\frac{t}{e^{-t}}$$

$$\overset{L'H}{=} \lim_{t\to-\infty}\frac{1}{-e^{-t}} = 0$$

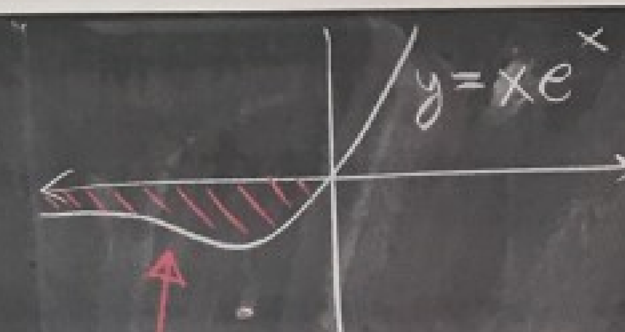
$\boxed{\text{"}\frac{-\infty}{\infty}\text{"}}$

$\boxed{\text{"}\frac{1}{-e^{\infty}} = \frac{1}{-\infty} = 0\text{"}}$

Therefore,

$$\int_{-\infty}^{0} xe^x\,dx = \lim_{t\to-\infty}\left(-te^t - 1 + e^t\right)$$

$$= 0 - 1 + 0 = \boxed{-1}$$



So, $\int_{-\infty}^{0} xe^x\,dx$ is convergent.

Or, it converges to -1.