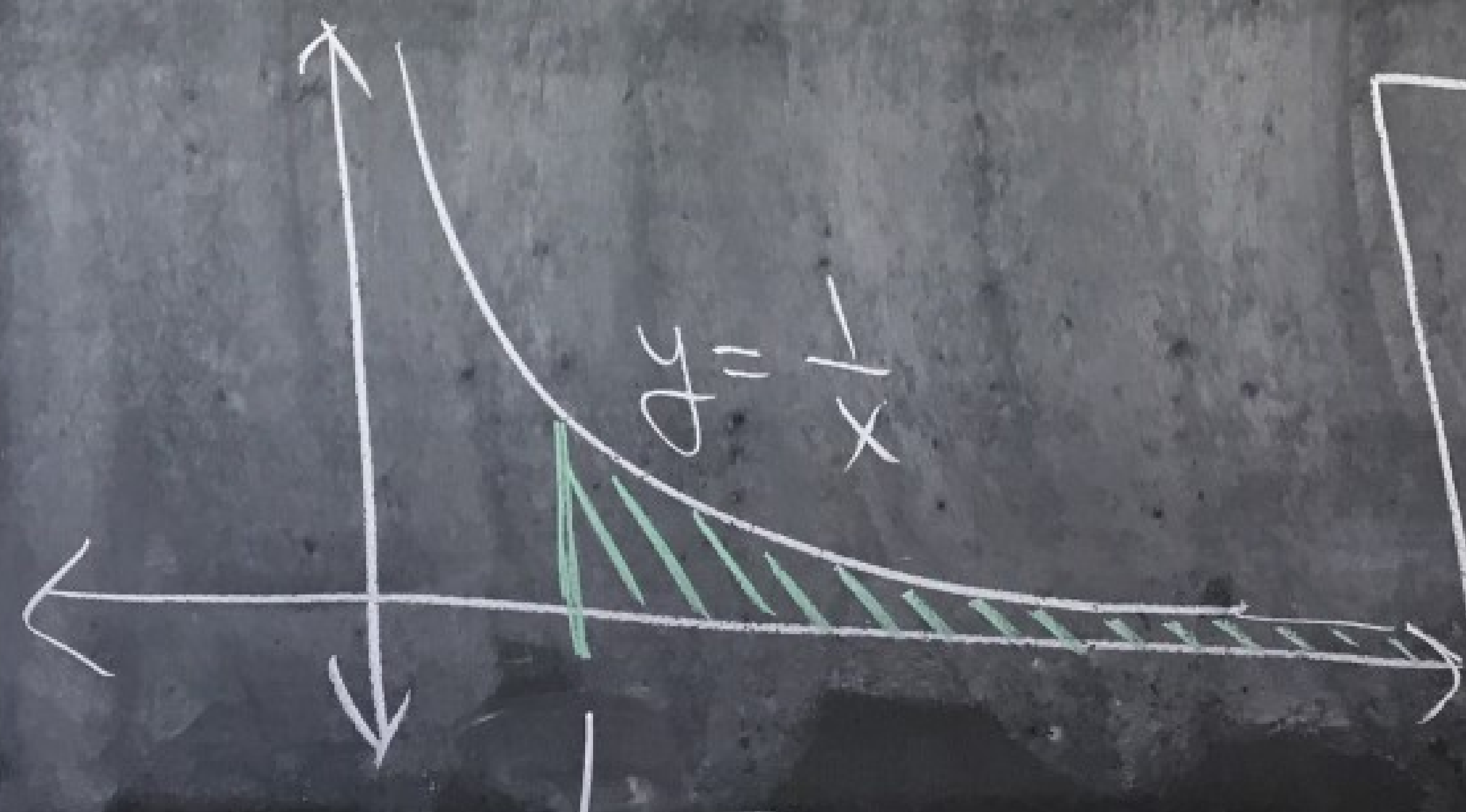
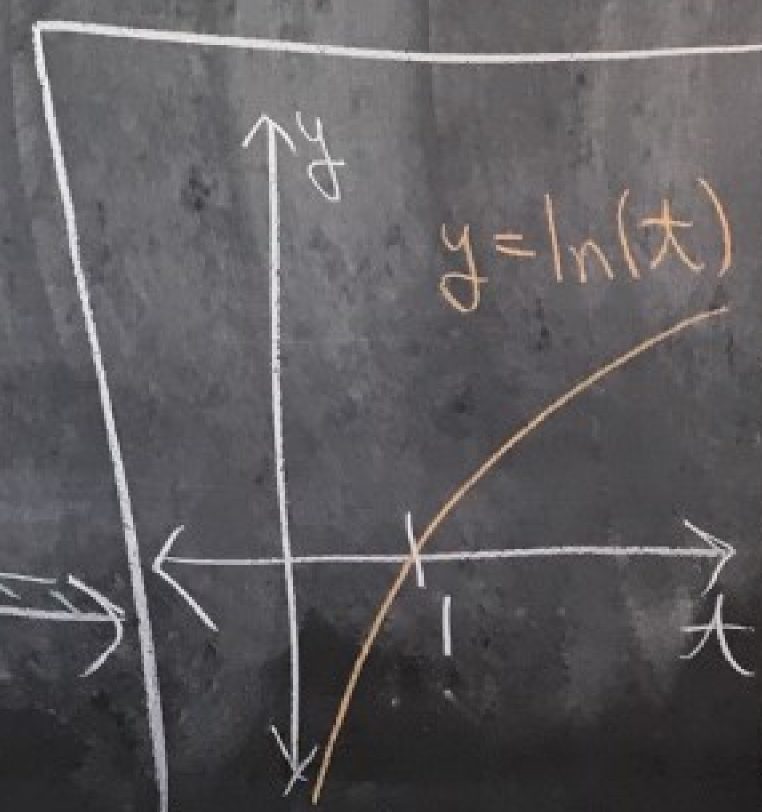
Weds
2/5

7.8 continued

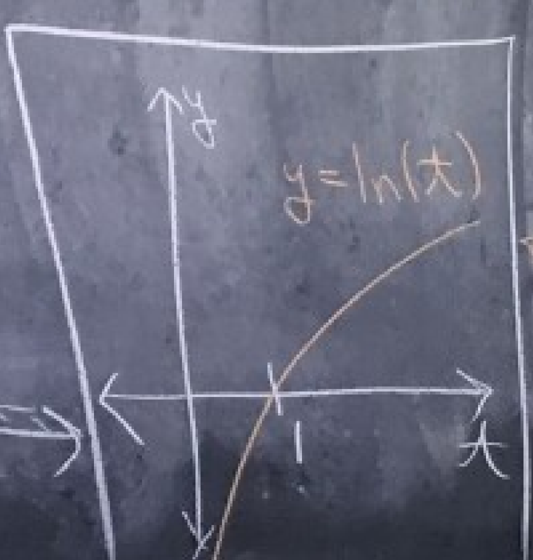
Ex: Does $\int_1^\infty \frac{1}{x}\,dx$ converge or diverge?

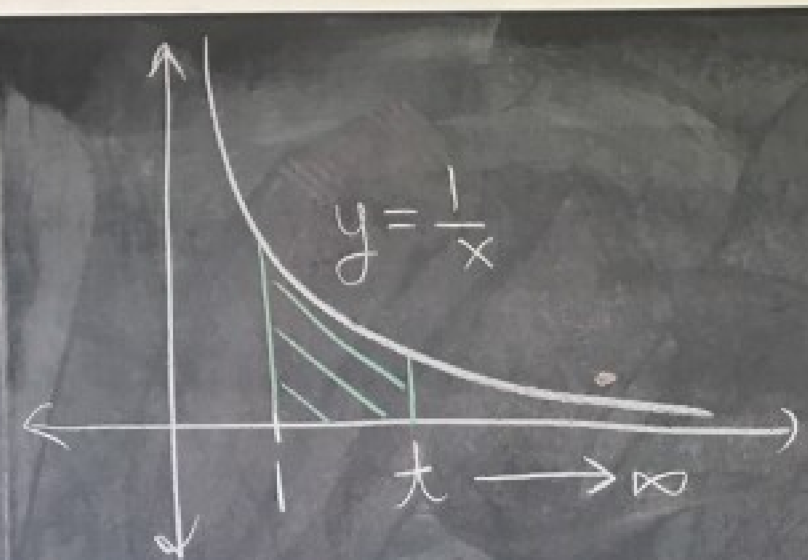




$\frac{1}{x}\,dx$

erge?





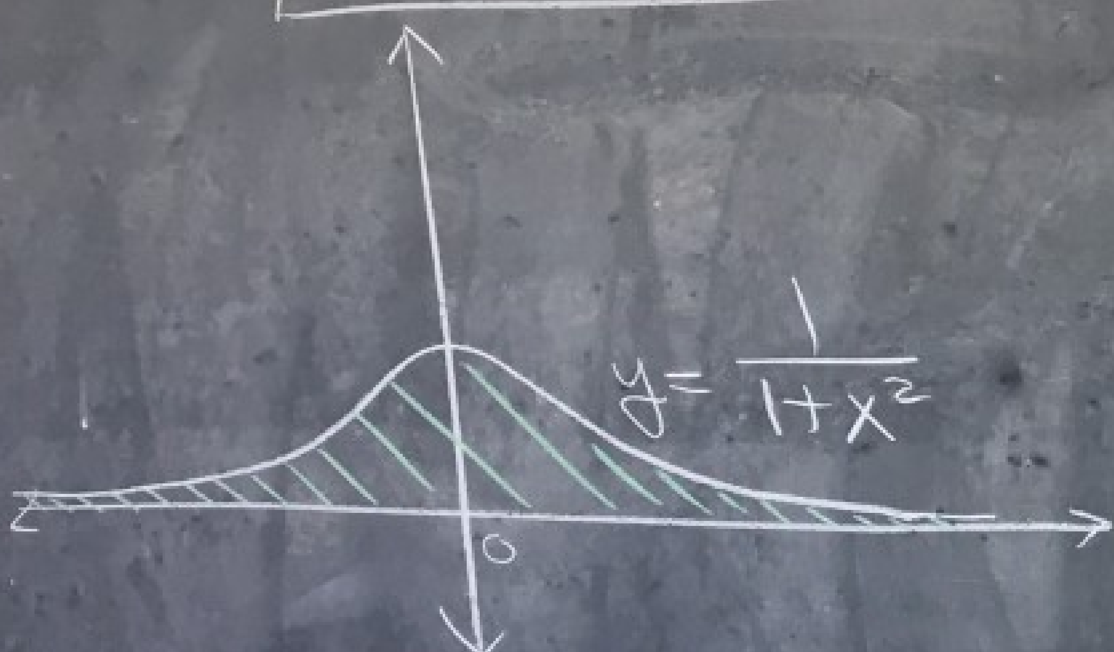
$$\lim_{t\to\infty}\int_1^t \frac{1}{x}\,dx = \lim_{t\to\infty}\left(\ln|x|\right)_1^t$$

$$= \lim_{t\to\infty}\left(\underbrace{\ln|t|}_{\to\infty} - \underbrace{\ln|1|}_{0}\right) = \infty$$

So, $\int_1^\infty \frac{1}{x}\,dx$

diverges.

Ex: Does $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ converge or diverge?



Pick any number a.
Let's pick $a=0$.
Then if $\displaystyle\int_{-\infty}^{0} \frac{dx}{1+x^2}$ and $\displaystyle\int_{0}^{\infty} \frac{dx}{1+x^2}$

converge then

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \int_{-\infty}^{0} \frac{dx}{1+x^2} + \int_{0}^{\infty} \frac{dx}{1+x^2}$$

an

If

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and so $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ converges.

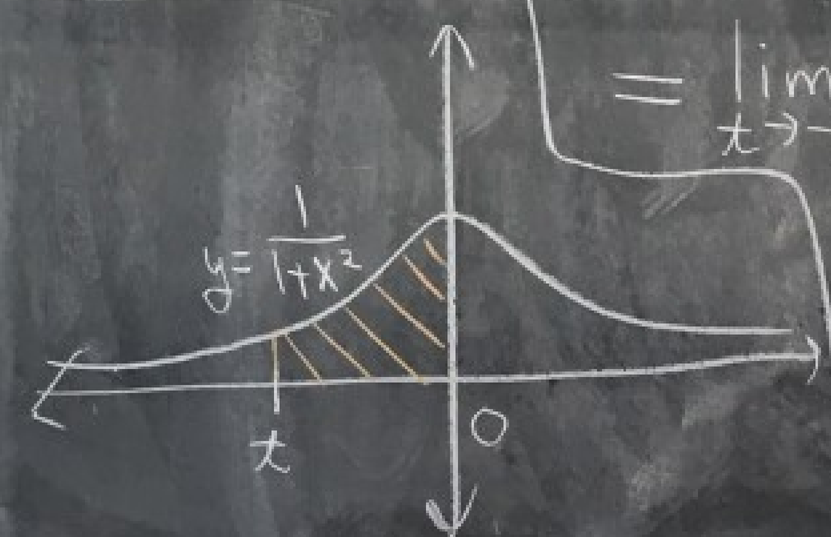
If either one or both of

$$\int_{-\infty}^{0} \frac{dx}{1+x^2} \quad \text{or} \quad \int_{0}^{\infty} \frac{dx}{1+x^2}$$

diverge then $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ diverges

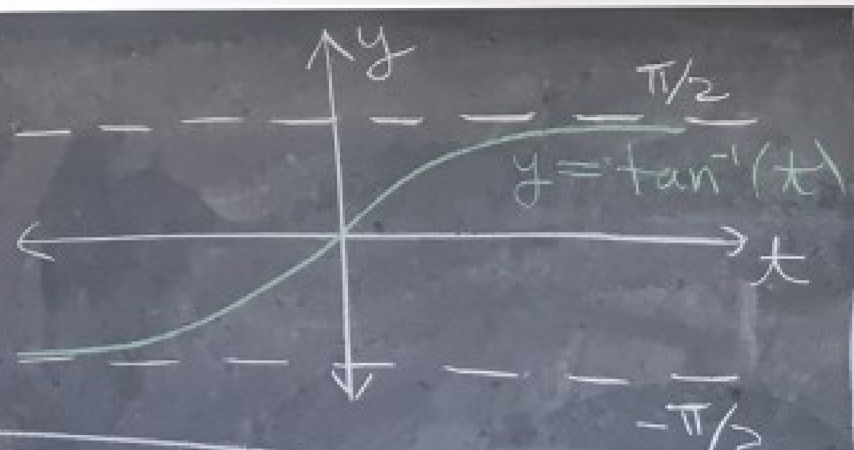
Let's calculate $\int_{-\infty}^{0} \frac{dx}{1+x^2}$.

$$\lim_{t\to-\infty} \int_{t}^{0} \frac{dx}{1+x^2} = \lim_{t\to-\infty} \left[\tan^{-1}(x)\right]_t^0$$



$$= \lim_{t\to-\infty} \left[\tan^{-1}(0) - \tan^{-1}(t)\right]$$

$$=$$



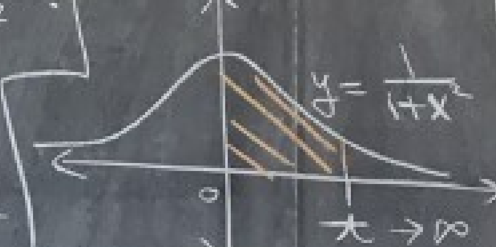
$$= \lim_{t\to-\infty}\left[0-\tan^{-1}(t)\right]$$

$$= \left[-\left(-\frac{\pi}{2}\right)\right] = \frac{\pi}{2}$$

So, $\displaystyle\int_{-\infty}^{0} \frac{dx}{1+x^2} = \frac{\pi}{2}$

Now let's calculate $\displaystyle\int_0^\infty \frac{dx}{1+x^2}$.

$$\int_0^\infty \frac{dx}{1+x^2} = \lim_{t\to\infty}\int_0^t \frac{dx}{1+x^2}$$



$$= \lim_{t\to\infty}\left(\tan^{-1}(x)\Big|_0^t\right) = \lim_{t\to\infty}\left(\tan^{-1}(t) - \underbrace{\tan^{-1}(0)}_{0}\right)$$

$$\lim_{t\to\infty}\tan^{-1}(t) = \frac{\pi}{2}.$$

Thus, $\displaystyle\int_0^\infty \frac{dx}{1+x^2} = \frac{\pi}{2}$.

Therefore,

$$\int_{-\infty}^{\infty}\frac{dx}{1+x^2} = \int_{-\infty}^{0}\frac{dx}{1+x^2} + \int_0^\infty \frac{dx}{1+x^2}$$

$$= \frac{\pi}{2} + \frac{\pi}{2} = \pi.$$

Def: (Improper integrals with discontinuous integrand)

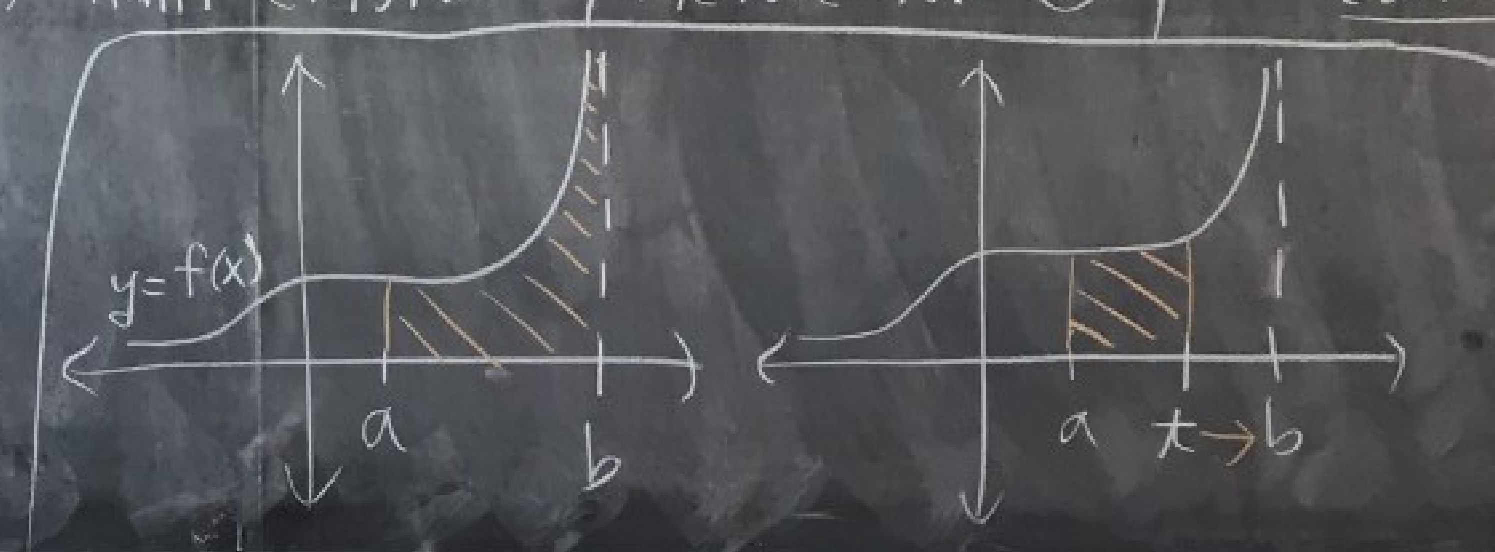
the function you're integrating

① If f is continuous on $[a,b)$ but discontinuous at b. Then define

$$\int_a^b f(x)\,dx = \lim_{t\to b^-}\int_a^t f(x)\,dx$$ if this limit exists.

If the limit exists we say that $\int_a^b f(x)\,dx$ converges, otherwise it diverges.

Picture for ①



② If f

discontinu

$\int_a^b f(x)\,d$

If the

converg

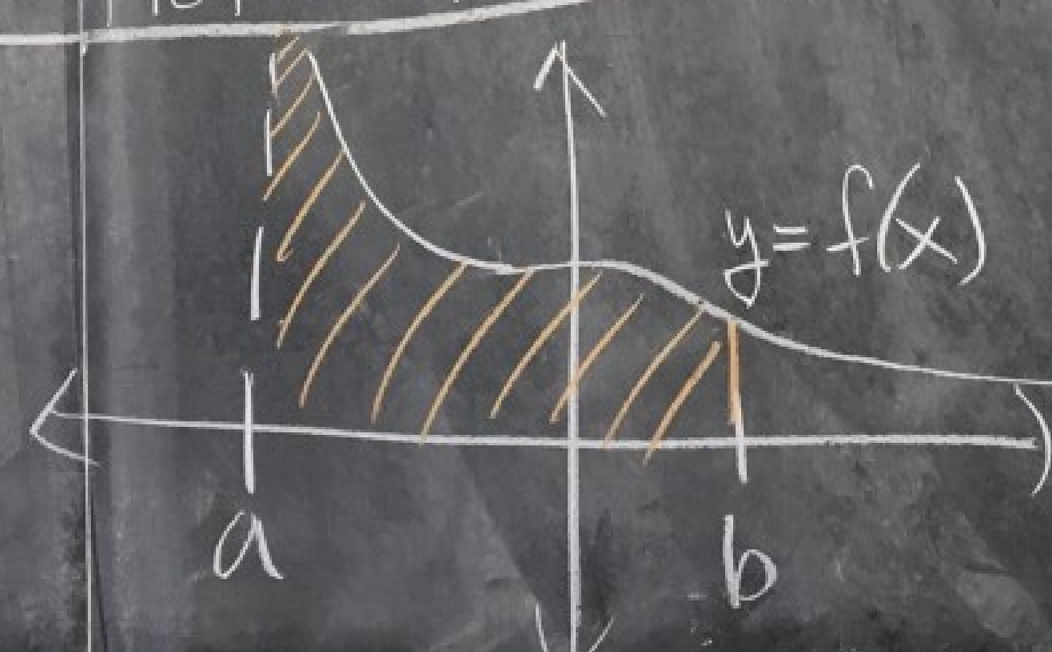
② If f is continuous on $(a, b]$ but discontinuous at a. Then define

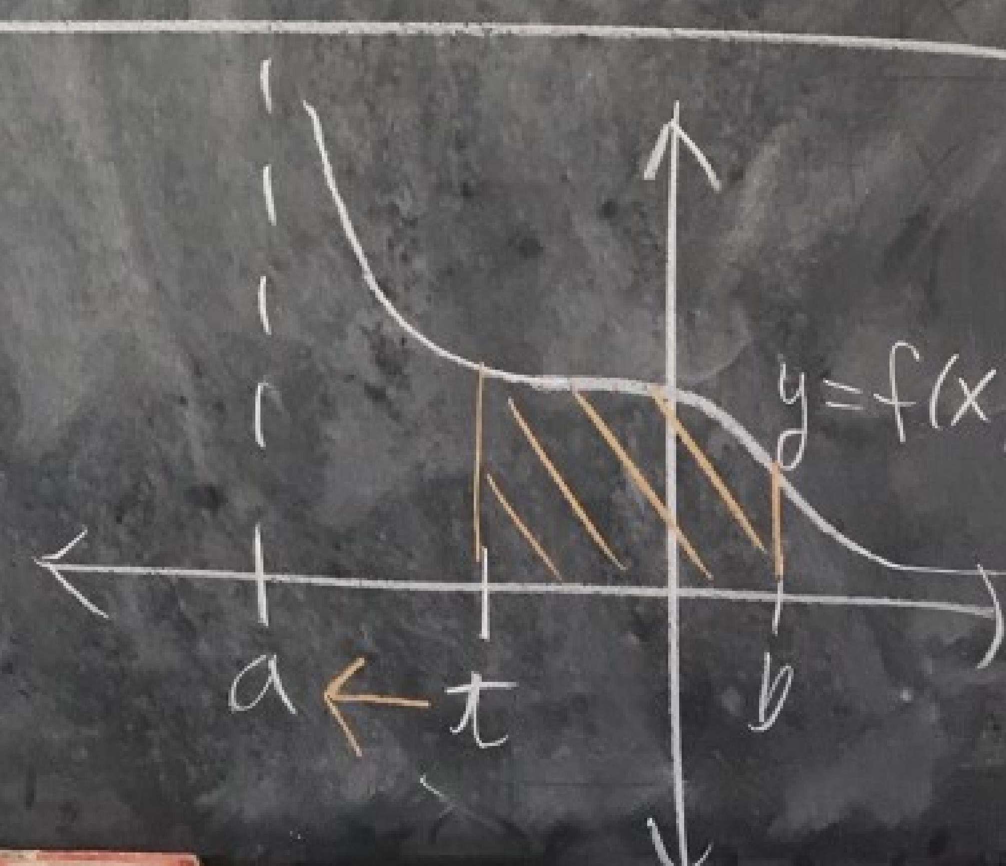
$$\int_a^b f(x)\,dx = \lim_{t \to a^+} \int_t^b f(x)\,dx$$ if this limit exists.

If the limit exists we say that $\int_a^b f(x)\,dx$ <u>converges</u>, otherwise it <u>diverges</u>.

r ①

Picture for ②





(3) If f has a discontinuity at c, where $a<c<b$, and both $\int_a^c f(x)\,dx$ and $\int_c^b f(x)\,dx$ converge

then define $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$