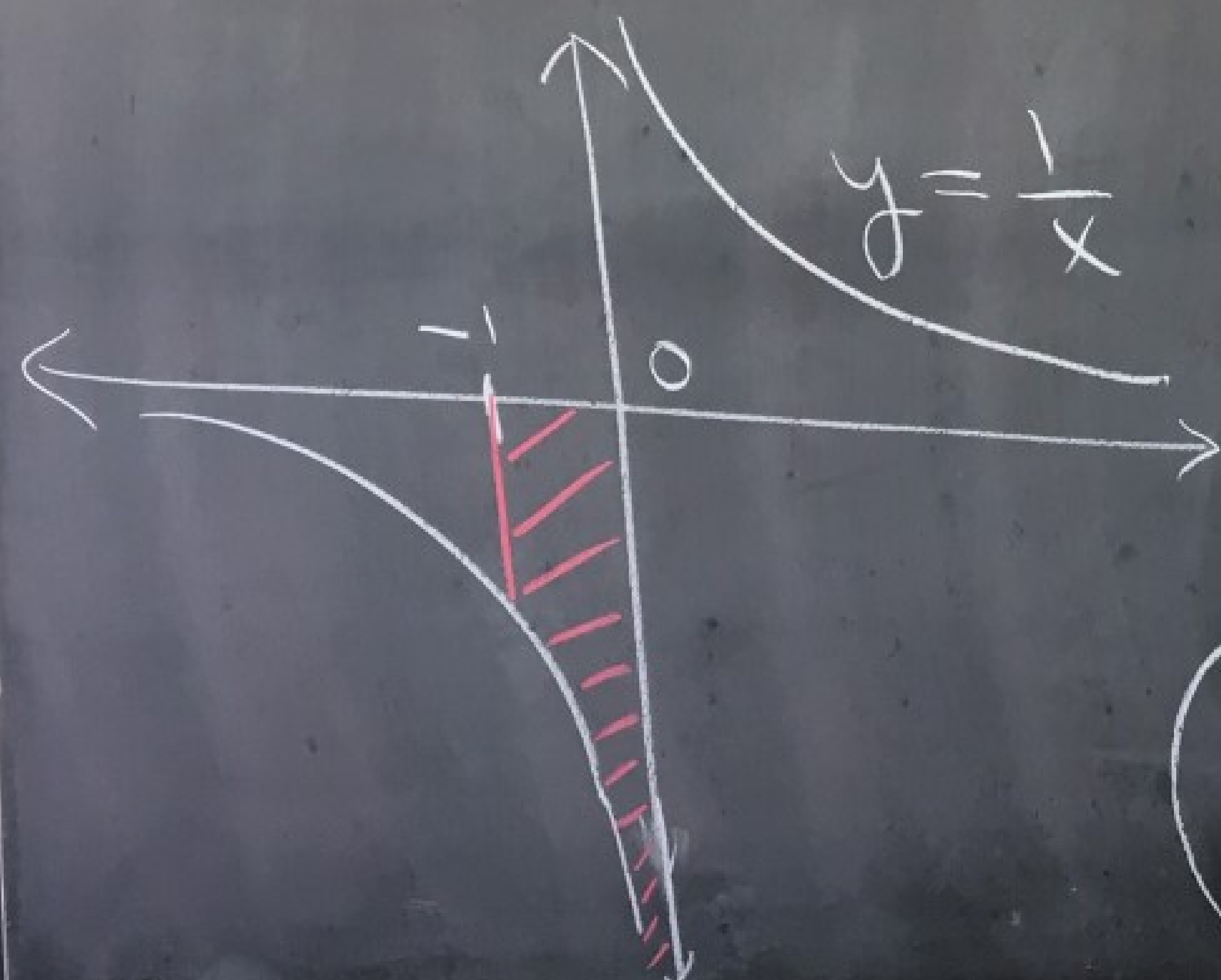
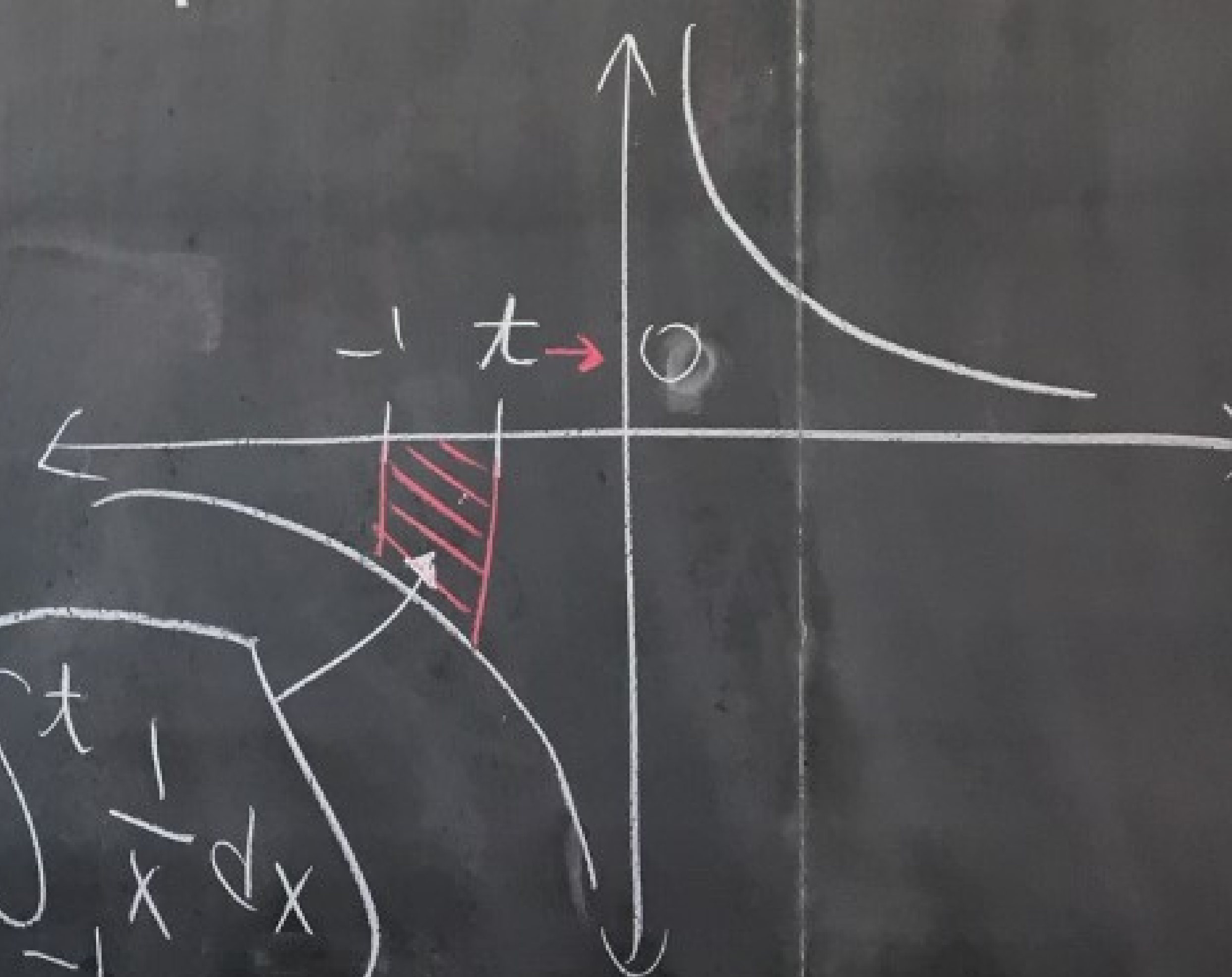
2/6
Thurs

(7.8 continued...)

Ex: Evaluate $\int_{-1}^{0} \frac{1}{x}\,dx$





$\int_{-1}^{t} \frac{1}{x}\,dx$

$\int_{-1}^{0}$

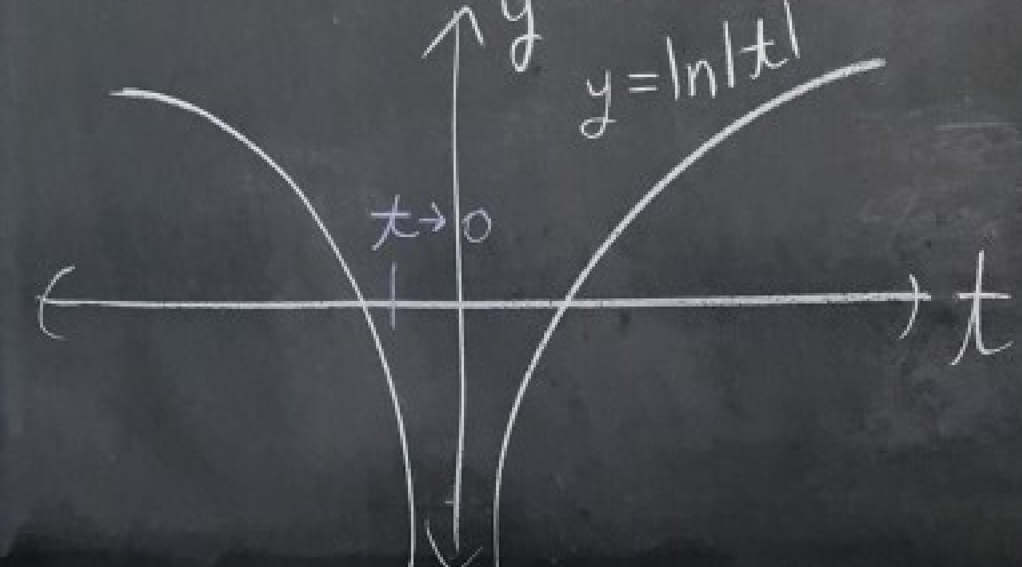
$=$

$$\int_{-1}^{0} \frac{1}{x}\,dx = \lim_{t\to 0^-} \int_{-1}^{t} \frac{1}{x}\,dx$$

$$= \lim_{t\to 0^-} \left[\ln|x| \Big|_{-1}^{t} \right]$$

$$\lim_{t\to 0^-} \left(\ln|t| - \underbrace{\ln|-1|}_{\ln(1)=0} \right)$$

$$= \lim_{t\to 0^-} \ln|t| = -\infty$$



So, $\int_{-1}^{0} \frac{1}{x}\,dx$

diverges.

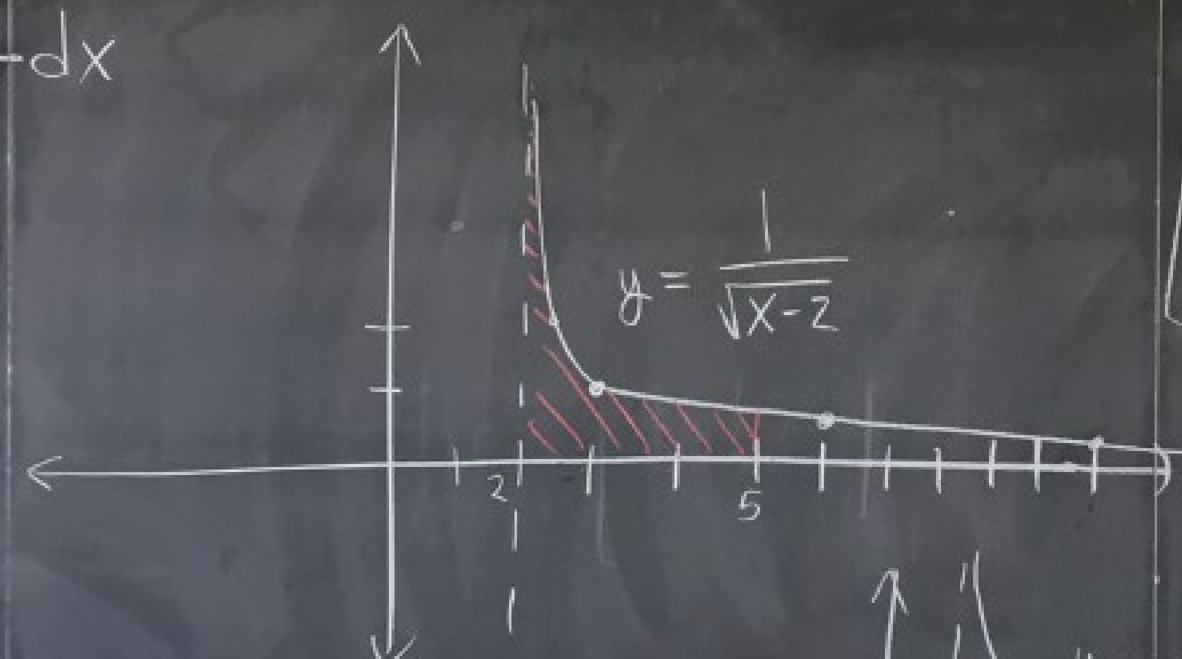
Ex: Evaluate $\int_2^5 \frac{1}{\sqrt{x-2}}\,dx$

picture of $\frac{1}{\sqrt{x-2}}$

You can only plot $x-2>0$.

Ie. $x>2$.

x	$\frac{1}{\sqrt{x-2}}$
3	1
6	$\frac{1}{2}$
11	$\frac{1}{3}$
$2+\frac{1}{4}$	$\frac{1}{\sqrt{\frac{1}{4}}}=2$



$$\int_2^5 \frac{dx}{\sqrt{x-2}} = \lim_{t\to 2^+}\int_t^5 \frac{dx}{\sqrt{x-2}}$$

$$= \lim_{t\to 2^+}\int_t^5 (x-2)^{-1/2}\,dx$$

$$= \lim_{t\to 2^+}\left[\frac{(x-2)^{1/2}}{1/2}\right]_t^5$$

$$= \lim_{t \to 2^+} \left[2\sqrt{5-2} - 2\sqrt{t-2} \right]$$

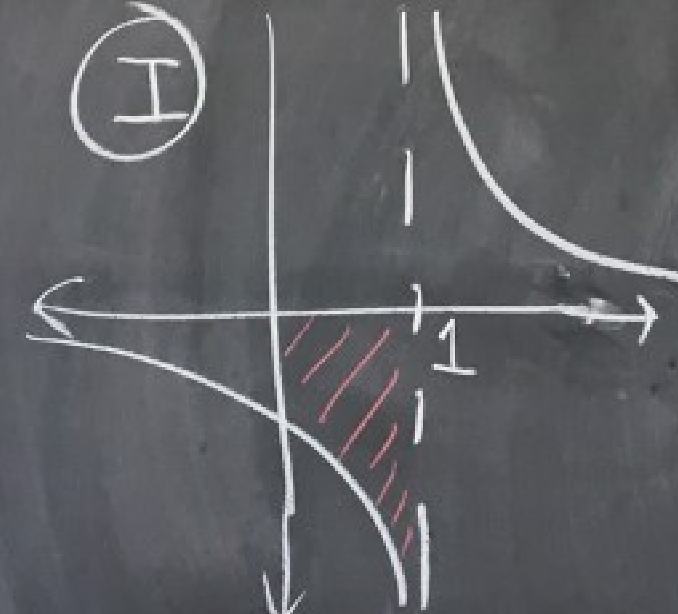
$$= 2\sqrt{3} - 2\sqrt{2-2}$$

$$= 2\sqrt{3} \approx 3.4641\ldots$$

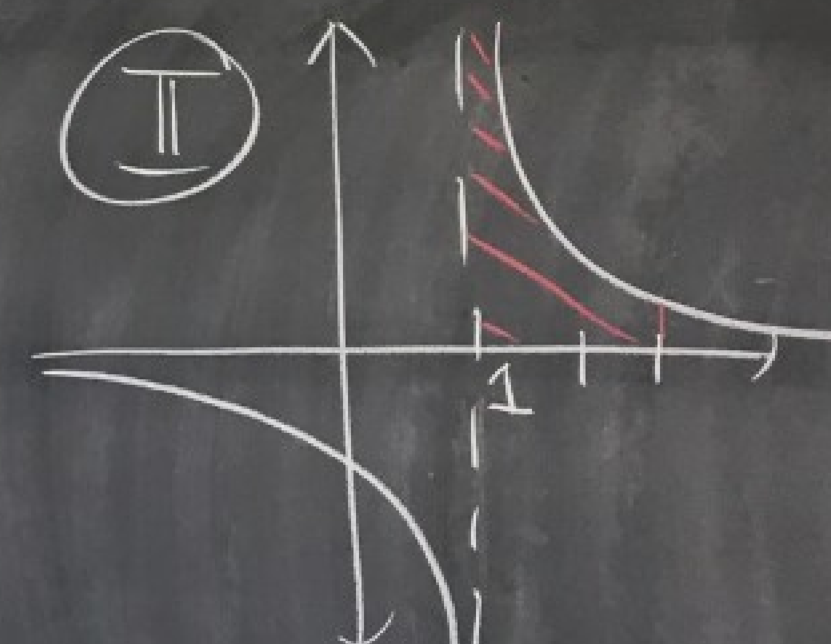
So, $\int_2^5 \frac{dx}{\sqrt{x-2}}$ converges to $2\sqrt{3}$

Ex: Evaluate $\int_0^3 \frac{dx}{x-1}$.

$=$ I



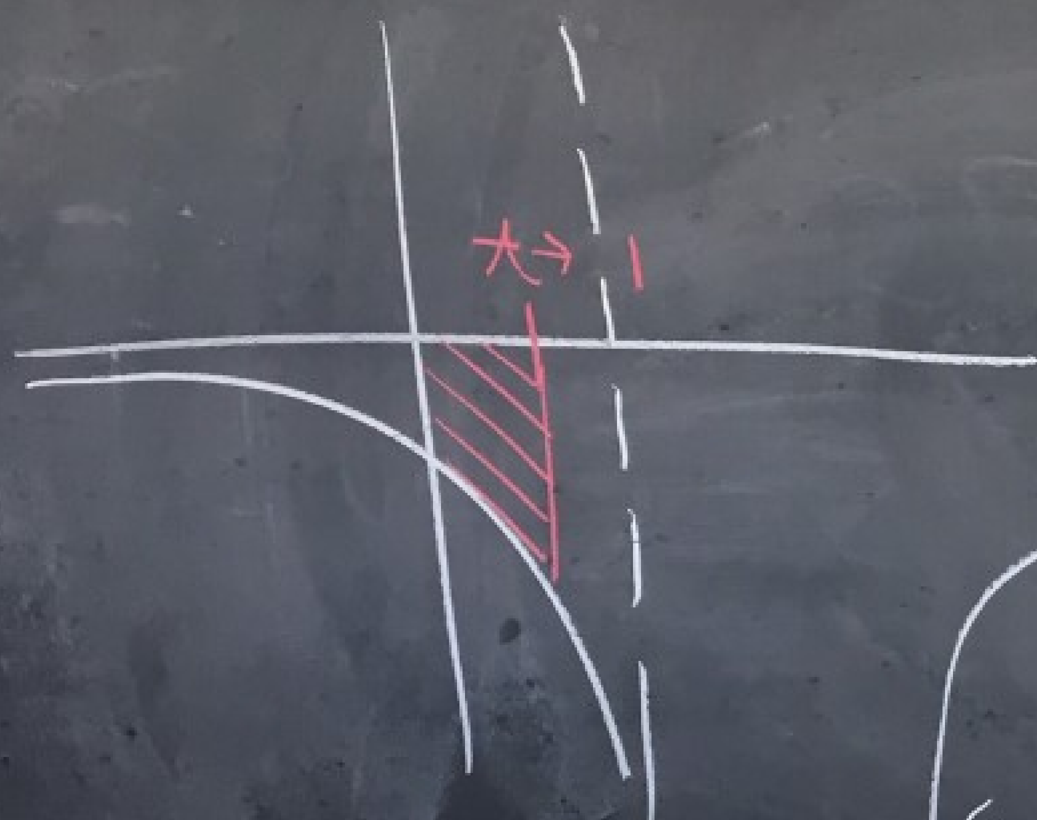
$+$ II



if both converge

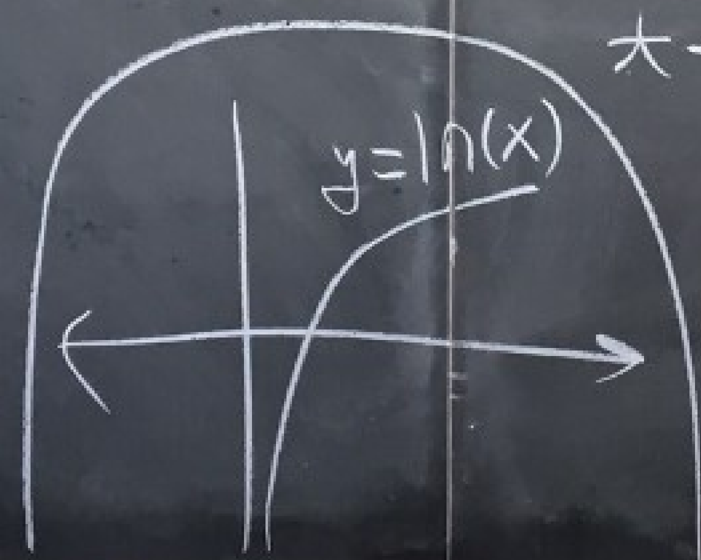
I

$$\lim_{t\to 1^-}\int_0^t \frac{dx}{x-1} = \lim_{t\to 1^-}\left(\ln|x-1|\right)\Big|_0^t$$



$$= \lim_{t\to 1^-} \ln|t-1| - \underbrace{\ln|-1|}_{\ln(1)=0}$$

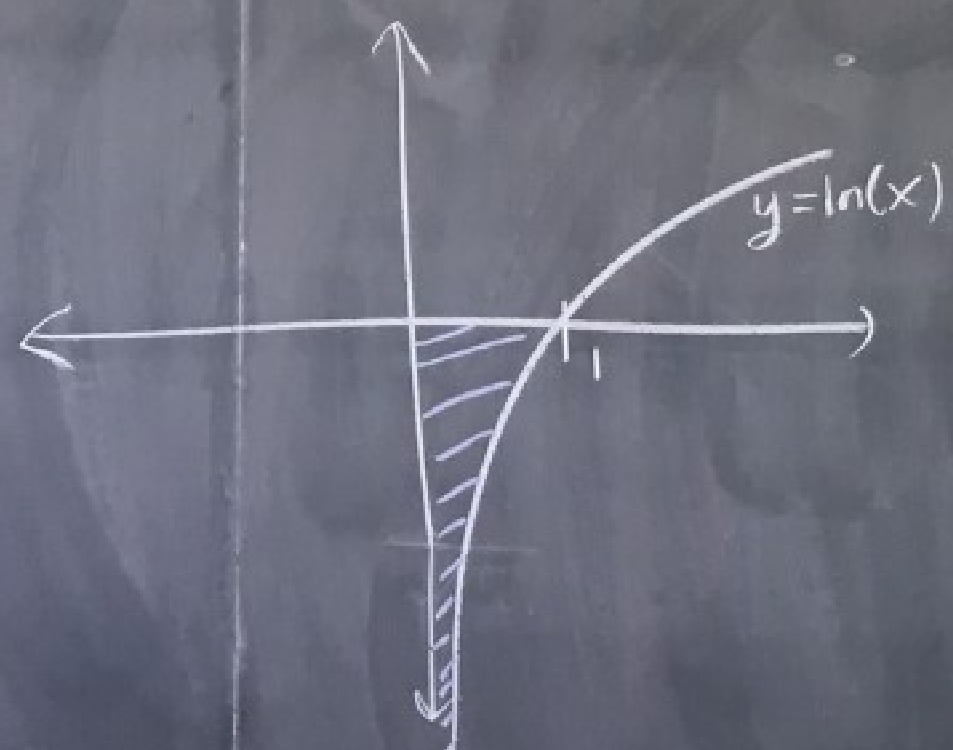
$$= \lim_{t\to 1^-} \ln|t-1| = -\infty$$

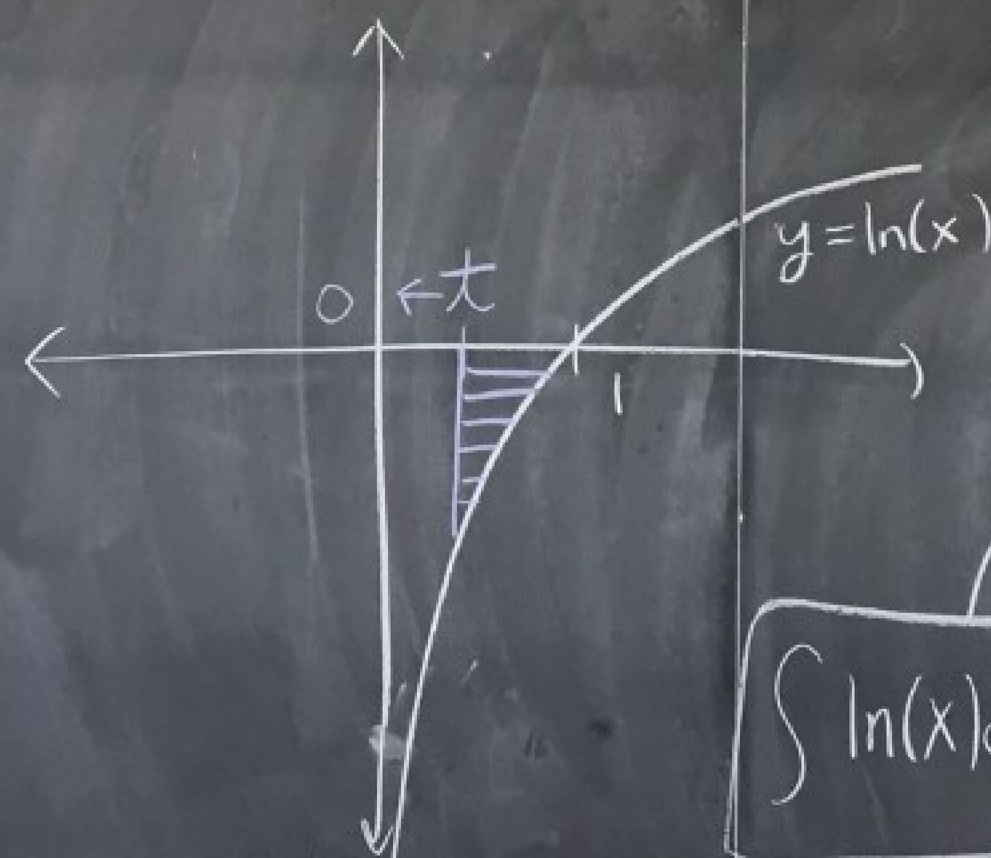


Answer: Since I diverges, $\int_0^3 \frac{dx}{x-1}$ diverges.

Ex

Ex: Evaluate $\int_0^1 \ln(x)\,dx$





$$\int_0^1 \ln(x)\,dx = \lim_{t\to 0^+} \int_t^1 \ln(x)\,dx$$

$$= \lim_{t\to 0^+} \Big[x\ln(x) - x \Big]_t^1$$

$$= \lim_{t\to 0^+} \Big[(1\cdot \ln(1) - 1) - (t\ln(t) - t) \Big]$$

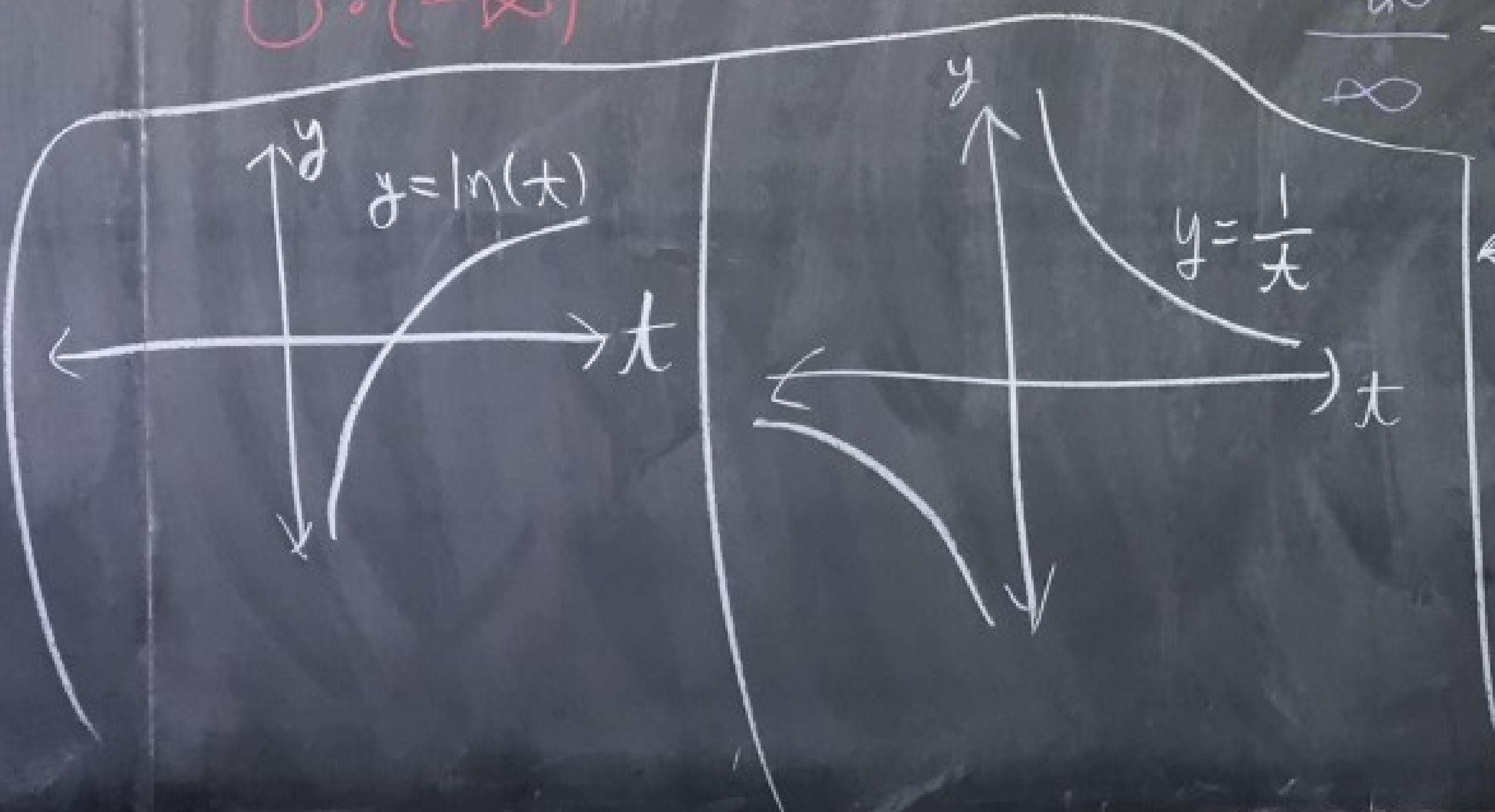
$$\boxed{\int \ln(x)\,dx = x\ln(x) - x + C}$$

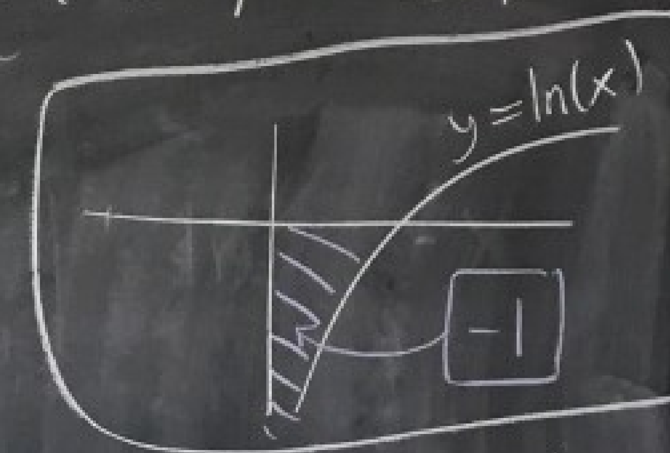
$$\underset{\substack{\uparrow \\ \boxed{\ln(1)=0}}}{=} \lim_{t\to 0^+}\left[-1 - t\ln(t) + \underset{\substack{\downarrow \\ 0}}{t}\right]$$

$$= -1 - \lim_{t\to 0^+}\left[t\ln(t)\right] + 0$$

Let's calculate $\lim_{t\to 0^+} t\ln(t)$

$$\lim_{t\to0^+} \underbrace{t\ln(t)}_{\text{"}0\cdot(-\infty)\text{"}} = \lim_{t\to0^+} \frac{\ln(t)}{\left(\frac{1}{t}\right)} \underset{\text{"}\frac{-\infty}{\infty}\text{"}}{\overset{L'H}{=}} \lim_{t\to0^+} \frac{1/t}{-1/t^2} = \lim_{t\to0^+}(-t) = 0.$$





So,

$$\int_0^1 \ln(x)\,dx = -1 - \lim_{t\to0^+}(t\ln(t))$$

$$= -1 - 0 = \boxed{-1}$$