Math 2120

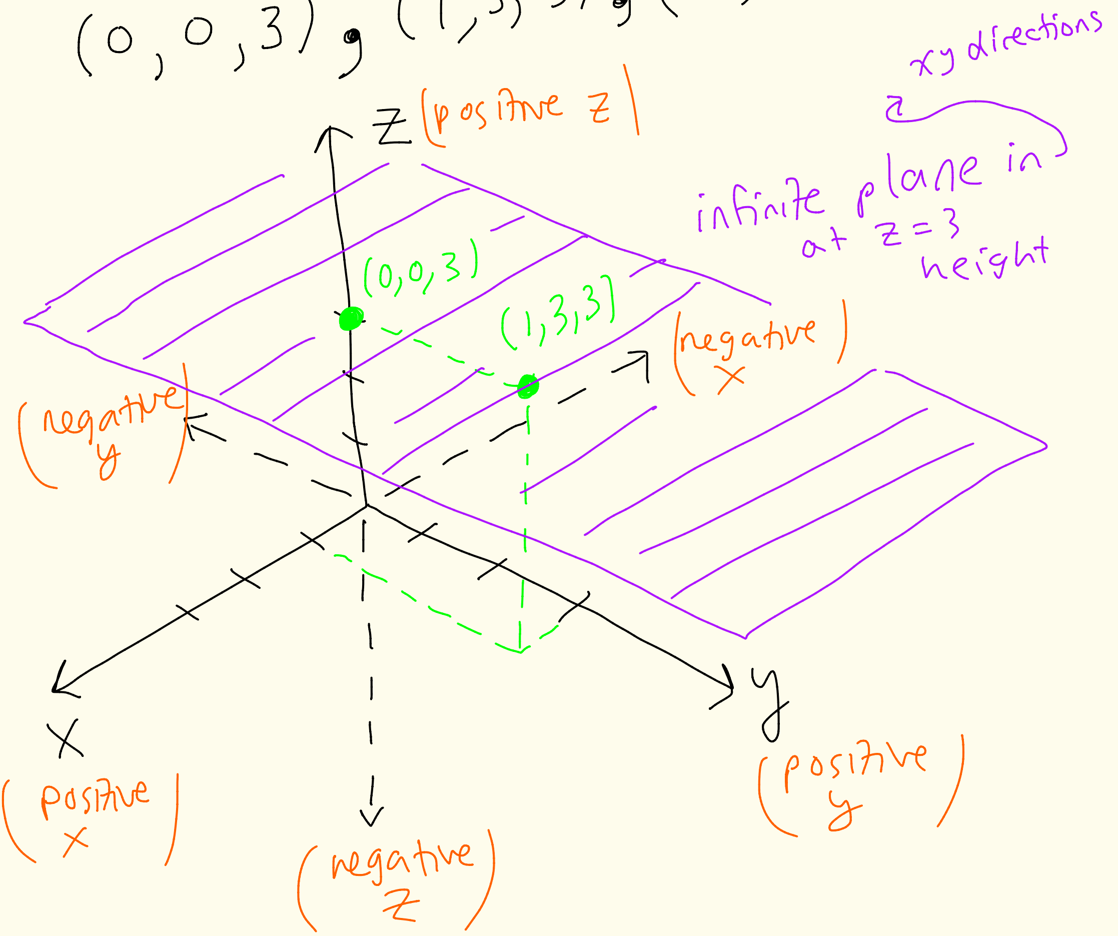
3/25/20

Weds

Ex: Sketch the surface
$z = 3$

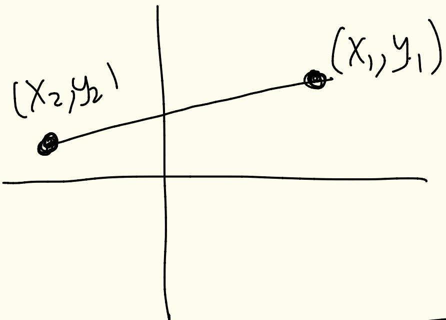
What are some points (x,y,z) that satisfy
$z = 3$?

$(0,0,3)$, $(1,3,3)$, $(-1,100,3)$, ...



Distance between two points (x_1, y_1) and (x_2, y_2) in 2d is

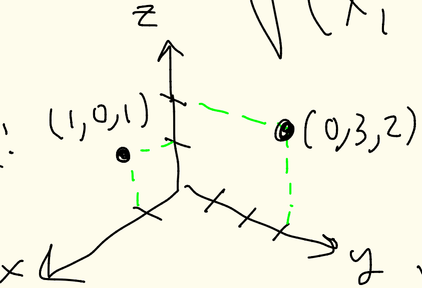
$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$



In 3d, the distance between (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Ex!

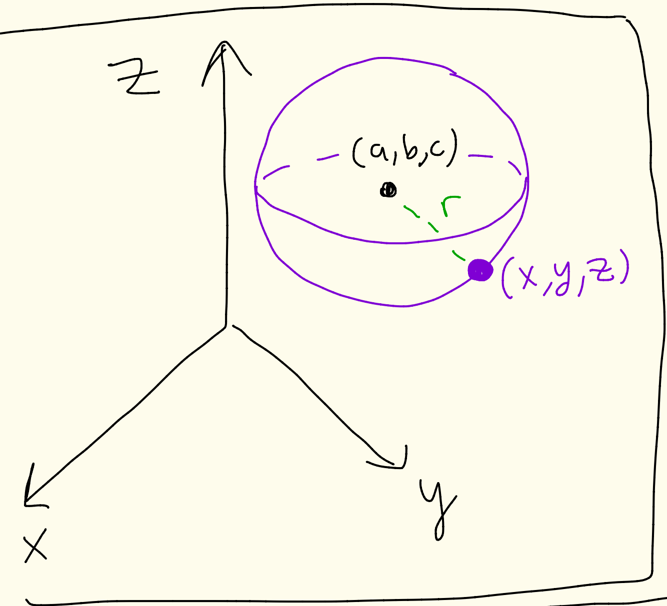


$\Leftarrow$ distance between (1,0,1) and (0,3,2) is

$$\sqrt{(1-0)^2 + (0-3)^2 + (1-2)^2} = \sqrt{11}$$

Eqn of a sphere

The equation of a sphere of radius r with center (a,b,c) will now be derived.



We want to find all (x,y,z) satisfying

$$\sqrt{(x-a)^2+(y-b)^2+(z-c)^2} = r$$

The formula for this sphere is

$$\boxed{(x-a)^2+(y-b)^2+(z-c)^2=r^2}$$

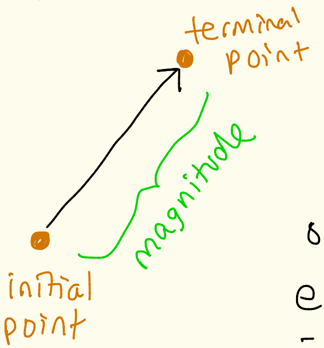
Sphere of radius 9 with center $(-1, 4, 7)$ is given by

$$(x-(-1))^2+(y-4)^2+(z-7)^2=9^2$$

$$\boxed{(x+1)^2+(y-4)^2+(z-7)^2=81}$$

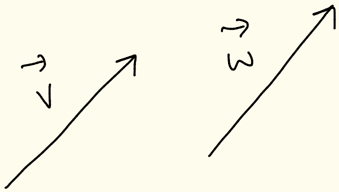
Vectors

The term vector is used to indicate a quantity (such as displacement, velocity, or force) that has both magnitude and direction. Vectors can be represented geometrically by arrows. The direction of the arrow specifies the direction of the vector and the length of the arrow describes its magnitude.

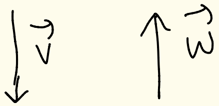


The tail (or starting point) of the vector is called the initial point of the vector, the tip (or ending point) of the vector is called the terminal point.

Two vectors $\vec{v}$ and $\vec{w}$ are considered equal (or equivalent) if they have the same length and direction. So it doesn't matter where you draw the vectors, ie you can move them around.



$\vec{v}$ and $\vec{w}$ are equivalent/equal

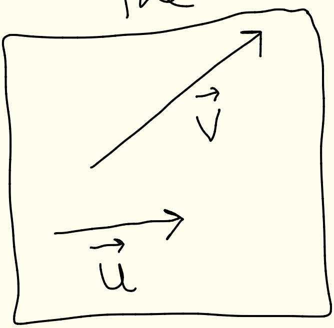


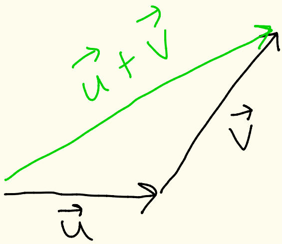
not equivalent/equal
same magnitude
but different directions

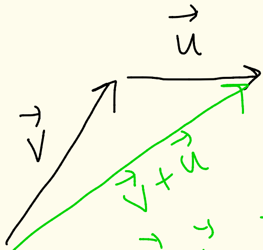
The <u>zero vector</u>, denoted by $\vec{0}$, has magnitude/length 0 and no specific direction. It's the only vector like this. It's draw as a dot

Vector addition

If $\vec{u}$ and $\vec{v}$ are vectors to calculate $\vec{u}+\vec{v}$ we position $\vec{v}$ so it's inital point is at the terminal point of $\vec{u}$, then $\vec{u}+\vec{v}$ is the vector with initial point taken from $\vec{u}$ and terminal point taken from $\vec{v}$. [Note: You can interchange $\vec{u}$ and $\vec{v}$ in this procedure and you get the same answer.]







Note: $\vec{u}+\vec{v}=\vec{v}+\vec{u}$

Workshop

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

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9.3 21 Find the first 4 nonzero terms of the Taylor series centered at a. Write the power series in summation notation.

$$f(x) = \sin(x), \quad a = \frac{\pi}{2}$$

$f(x) = \sin(x)$ $\quad f^{(0)}\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

$f'(x) = \cos(x)$ $\quad f^{(1)}\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

$f^{(2)}(x) = -\sin(x)$ $\quad f^{(2)}\left(\frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{2}\right) = -1$

$f^{(3)}(x) = -\cos(x)$ $\quad f^{(3)}\left(\frac{\pi}{2}\right) = -\cos\left(\frac{\pi}{2}\right) = 0$

$\vdots$ $\vdots$ repeats $\qquad \vdots$ $\vdots$ repeats

$$\frac{1}{0!}\left(x-\frac{\pi}{2}\right)^0 + \frac{0}{1!}\left(x-\frac{\pi}{2}\right)^1 + \frac{-1}{2!}\left(x-\frac{\pi}{2}\right)^2$$

$$+ \frac{0}{3!}\left(x-\frac{\pi}{2}\right)^3 + \frac{1}{4!}\left(x-\frac{\pi}{2}\right)^4 + \frac{0}{5!}\left(x-\frac{\pi}{2}\right)^5 + \frac{-1}{6!}\left(x-\frac{\pi}{2}\right)^6$$

$$+ \cdots$$

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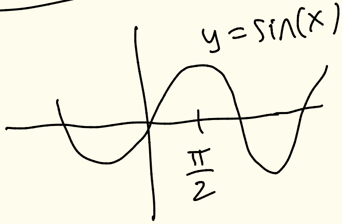
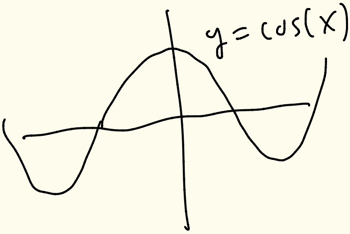
$$= 1 - \frac{1}{2!}\left(x-\frac{\pi}{2}\right)^2 + \frac{1}{4!}\left(x-\frac{\pi}{2}\right)^4$$

$$- \frac{1}{6!}\left(x-\frac{\pi}{2}\right)^6 + \frac{1}{8!}\left(x-\frac{\pi}{2}\right)^8 - \dots$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}\left(x-\frac{\pi}{2}\right)^{2k}$$





even	odd
$2k$	$2k+1$
	$2k-1$

$\boxed{9.3 \mid 29}$ pg 10

Find the first 4 nonzero terms in the Taylor series centered at $a=0$ [Maclaurin series] for $\ln(1+x^2)$.

Know: $\frac{d}{dx}\ln(1+x^2) = \frac{2x}{1+x^2}$

So, $\int \frac{2x}{1+x^2}\,dx = \ln(1+x^2) + C$

$r = -x^2$

$|-x^2| < 1$

$|x|^2 < 1$

$|x| < 1$

$$\frac{2x}{1+x^2} = 2x\left[\frac{1}{1-(-x^2)}\right] = 2x\sum_{k=0}^{\infty}(-x^2)^k$$

$$\sum_{k=0}^{\infty} r^k = 1 + r + r^2 + \dots = \frac{1}{1-r}$$

$-1 < r < 1$

$|r| < 1$

$$= 2x\sum_{k=0}^{\infty}(-1)^k x^{2k}$$

$$= \boxed{\sum_{k=0}^{\infty} 2(-1)^k x^{2k+1}}$$

$$\frac{2x}{1+x^2} = \sum_{k=0}^{\infty} 2(-1)^k x^{2k+1}$$

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$-1 < x < 1$ converges

When we integrate we will still get the same radius of convergence but the endpoints might converge.

$$\ln(1+x^2) = C + \int \frac{2x}{1+x^2}\,dx = C + \int \sum_{k=0}^{\infty} 2(-1)^k x^{2k+1}\,dx$$

$$= C + \sum_{k=0}^{\infty} 2(-1)^k \frac{x^{2k+2}}{2k+2}$$

$$= C + \underbrace{\tfrac{2}{2}}_{k=0} x^2 - \underbrace{\tfrac{2}{4}}_{k=1} x^4 + \underbrace{\tfrac{2}{6}}_{k=2} x^6 - \tfrac{2}{8} x^8 + \dots$$

Plug in $x=0$

$$\ln(1) = C + \underbrace{\tfrac{2}{2}0^2 - \tfrac{2}{4}0^4 + \tfrac{2}{6}0^6 - \dots}_{0}$$

so, $C = \ln(1) = 0$

$$\ln(1+x^2) = \sum_{k=0}^{\infty} 2(-1)^k \frac{x^{2k+2}}{2k+2}$$

converges $-1 < x < 1$

$$\ln(1+x^2) = \sum_{k=0}^{\infty} \frac{2(-1)^k}{2k+2} x^{2k+2} \qquad \boxed{-1<x<1}$$

check endpoints:

$\boxed{x=-1}$

$$\sum_{k=0}^{\infty} \frac{2(-1)^k}{2k+2} \underbrace{(-1)^{2k+2}}_{1} = \sum_{k=0}^{\infty} (-1)^k \frac{2}{2k+2}$$

even

converge by alt. series test

$\boxed{x=1}$

$$\sum_{k=0}^{\infty} \frac{2(-1)^k}{2k+2} (1)^{2k+2} = \sum_{k=0}^{\infty} (-1)^k \frac{2}{2k+2}$$

So,

$$\boxed{\ln(1+x^2) = \sum_{k=0}^{\infty} \frac{2(-1)^k}{2k+2} x^{2k+2} \qquad -1 \le x \le 1}$$