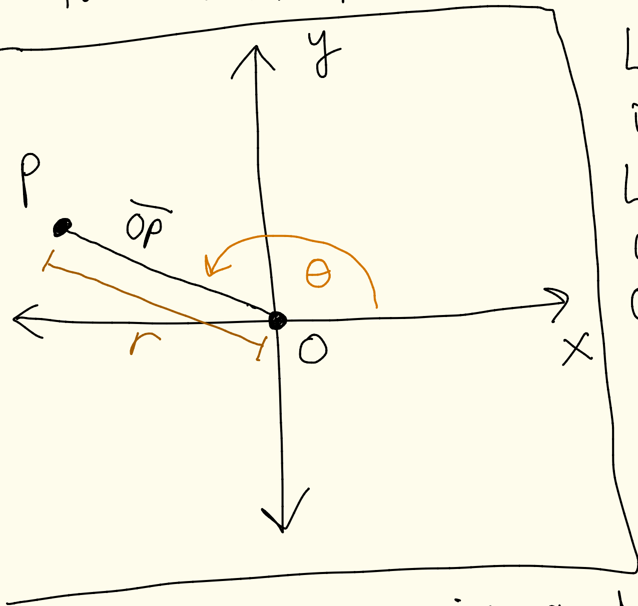
Math 2120

4/16/20

10.2 - Polar Coordinates

Polar coordinates are another way to label points in the xy-plane.

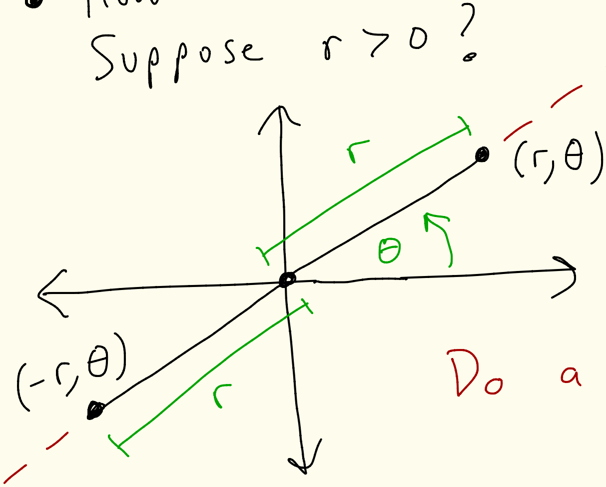


Let P be a point in the xy-plane. Let O be the origin. Connect O and P to get the line segment $\overline{OP}$. Let θ be the angle between the positive x-axis and $\overline{OP}$. Let r be the length of $\overline{OP}$. The pair (r, θ) are called <u>polar coordinates</u> for P.

<u>Conventions:</u>

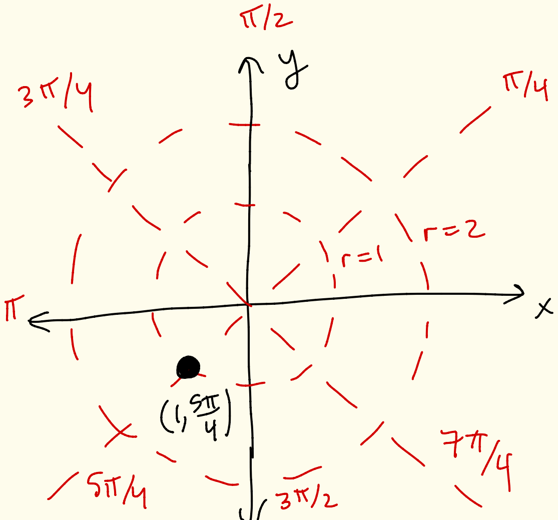
- The angle θ is positive if measured in the counterclockwise direction, and negative in the clockwise direction.

- If $P = O = (0,0)$, then $r = 0$ and θ can be any angle.
- How do we graph negative r? Suppose $r > 0$?

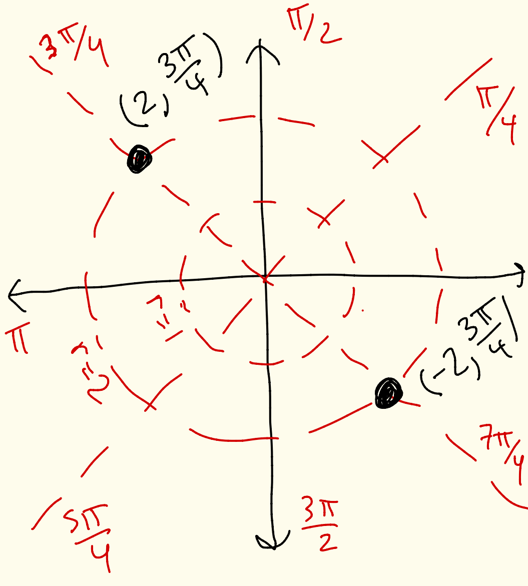


Do a 180° degree flip.

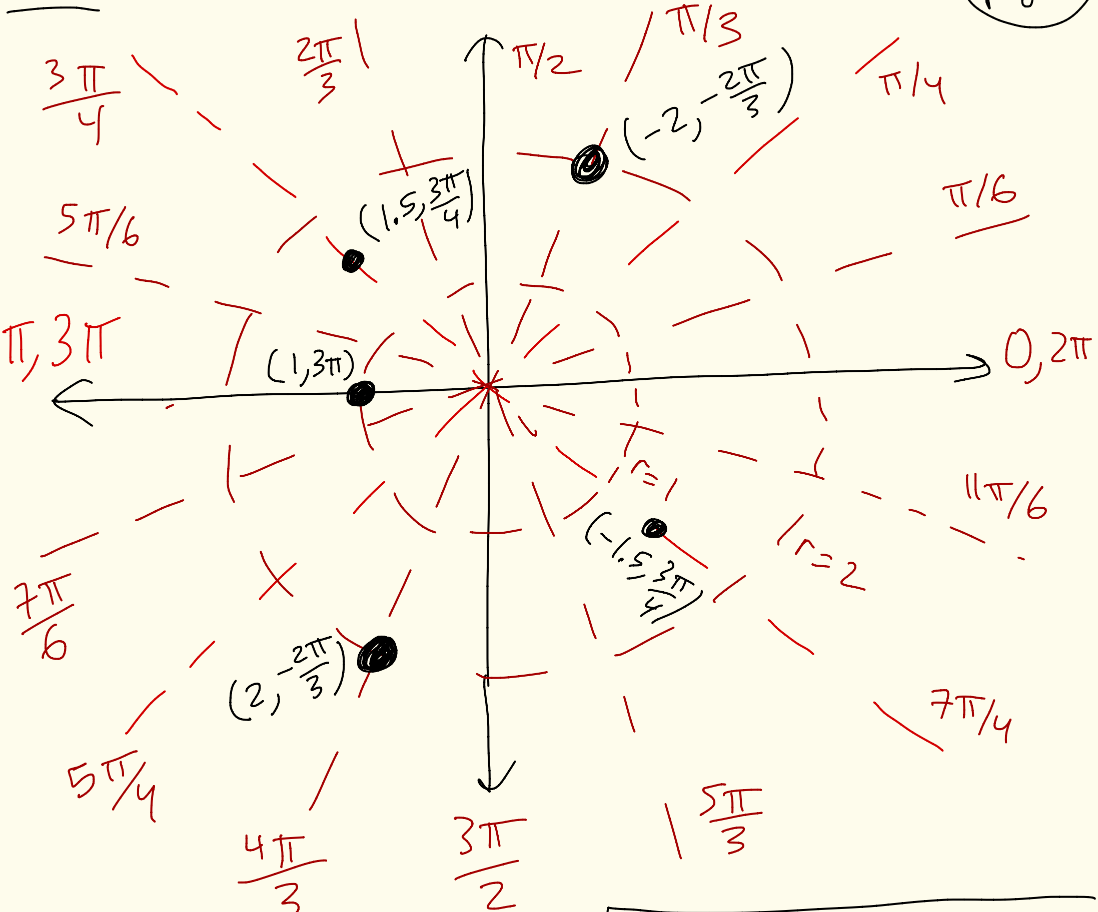
Ex: Plot $(r,\theta) = (1, \frac{5\pi}{4})$



Ex: Plot $(r,\theta) = (-2, \frac{3\pi}{4})$



Ex!

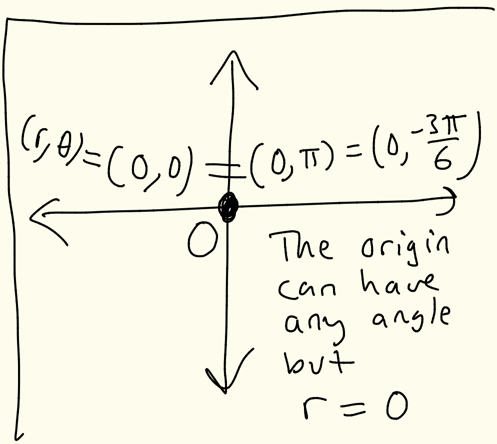


$(r, \theta) = (1, 3\pi)$

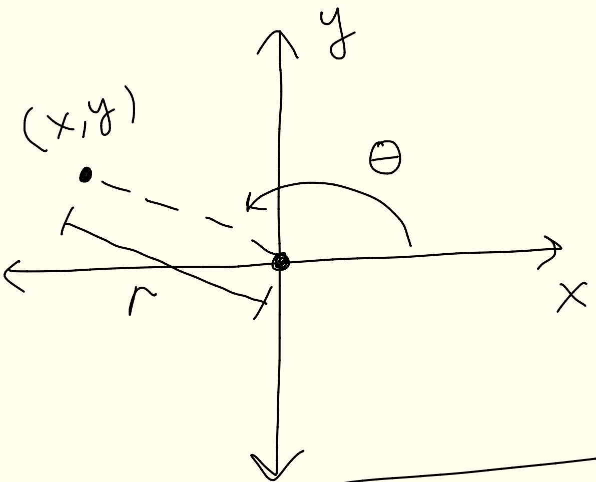
$(r, \theta) = \left(2, -\frac{2\pi}{3}\right)$

$(r, \theta) = \left(-2, -\frac{2\pi}{3}\right)$

$(r, \theta) = \left(-1.5, \frac{3\pi}{4}\right)$

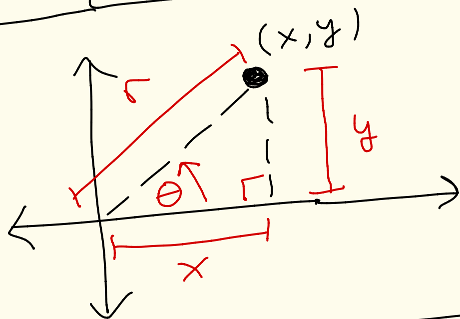


Relationship between Polar and Cartesian (xy) coordinates



$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$
$$x^2 + y^2 = r^2$$
$$\tan(\theta) = \frac{y}{x}$$

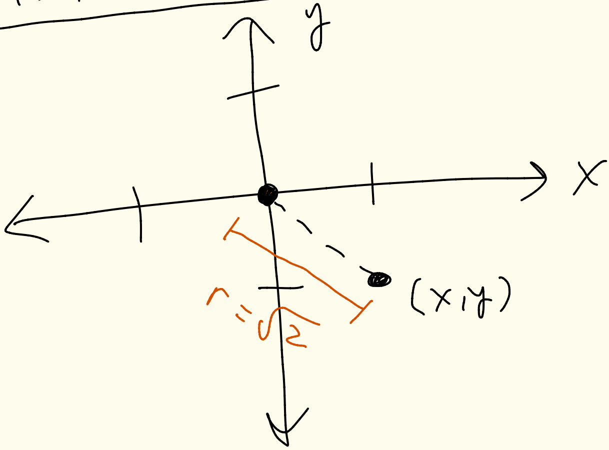
How to remember:



Ex: Convert $(r, \theta) = (2, \frac{\pi}{3})$ to Cartesian coordinates.

$$x = r\cos(\theta) = 2\cos\left(\frac{\pi}{3}\right) = 2\left(\frac{1}{2}\right) = 1$$
$$y = r\sin(\theta) = 2\sin\left(\frac{\pi}{3}\right) = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$
$$(x, y) = (1, \sqrt{3})$$

Ex: Convert $(x,y) = (1,-1)$ into polar coordinates.



$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

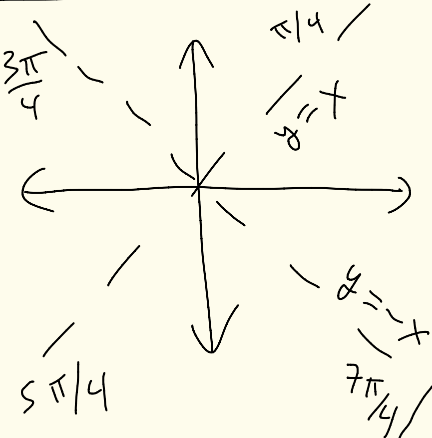
$$= \sqrt{1^2 + (-1)^2}$$

$$= \sqrt{2}$$

$$\tan(\theta) = \frac{y}{x}$$

$$\tan(\theta) = \frac{-1}{1} = -1$$

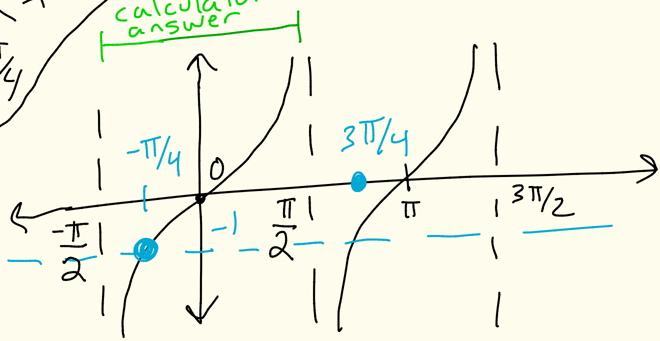
$$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$$



good short-cut

Calculator

$$\tan^{-1}(-1) = -\frac{\pi}{4}$$



Answer

$$(r, \theta) = \left(\sqrt{2}, -\frac{\pi}{4}\right)$$