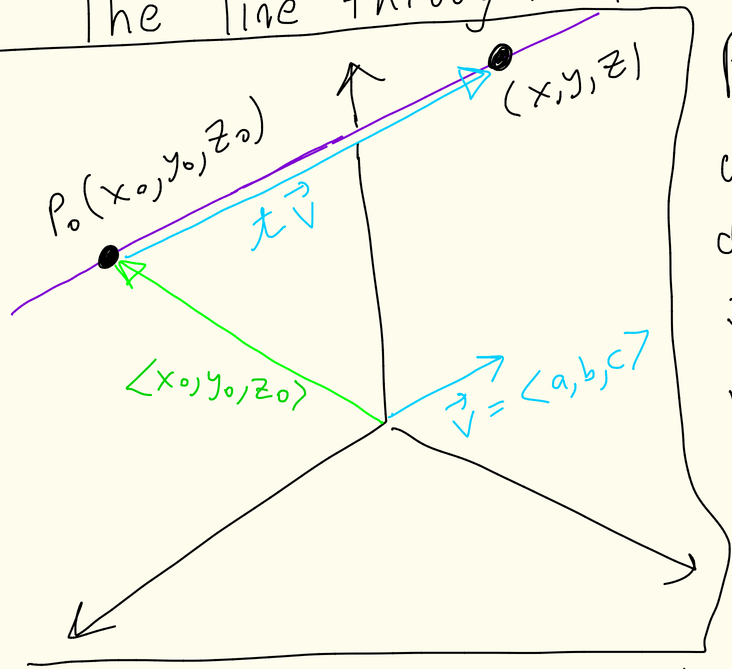
Math 2120

4/23/20

11.5 continued...

Last time:

The line through the point $P_0(x_0, y_0, z_0)$ and in the direction of $\vec{v} = \langle a, b, c \rangle$ is given by



$$\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t\vec{v}$$
$$= \langle x_0 + ta,\ y_0 + tb,\ z_0 + tc \rangle$$

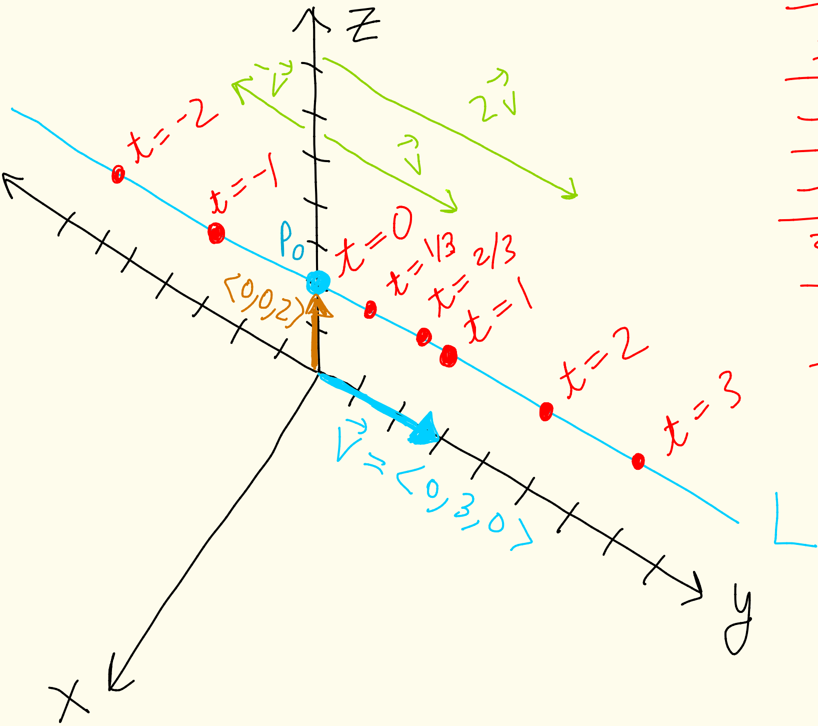
$$x = x_0 + ta$$
$$y = y_0 + tb$$
$$z = z_0 + tc$$

t is any real number

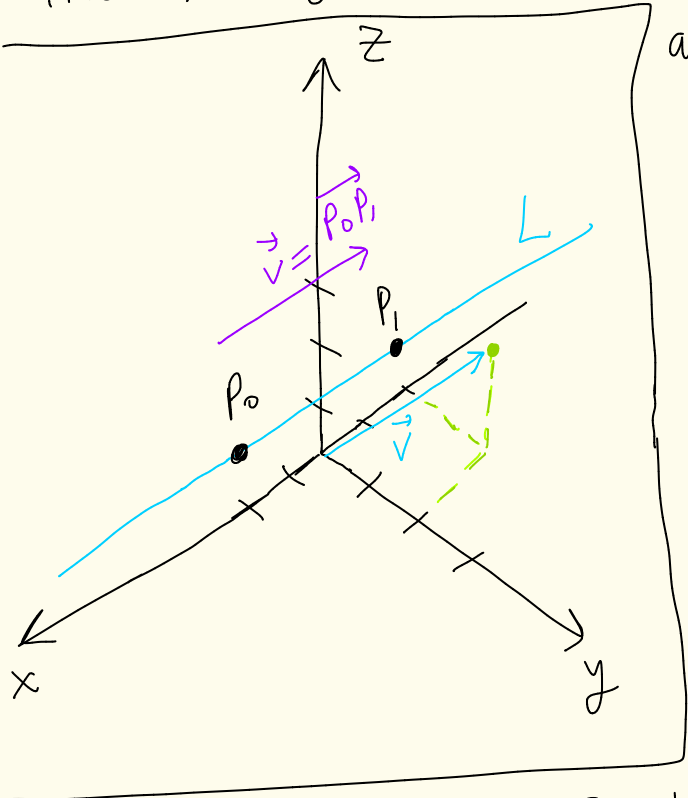
Ex: Find an equation for the line L through $P_0(0,0,2)$ in the direction of $\vec{v} = \langle 0,3,0 \rangle$.

$$x = 0 + 0t = 0$$
$$y = 0 + 3t = 3t$$
$$z = 2 + 0t = 2$$

t	(x,y,z)
0	(0,0,2)
1	(0,3,2)
2	(0,6,2)
3	(0,9,2)
−1	(0,−3,2)
−2	(0,−6,2)
2/3	(0,2,2)
1/3	(0,1,2)



Ex: Find an equation for the line through the points $P_0(2,0,1)$ and $P_1(0,2,3)$.



We need a vector in the direction of the line.

How about $\overrightarrow{P_0P_1}$?

$\overrightarrow{P_0P_1} = \langle 0-2, 2-0, 3-1 \rangle = \langle -2, 2, 2 \rangle$

Let's use $P_0(2,0,1)$ as the point on the line.

$$\boxed{\begin{aligned} x &= 2-2t = 2-2t \\ y &= 0+2t = 2t \\ z &= 1+2t = 1+2t \end{aligned} \quad t \text{ any real number}}$$

11.6 - Calculus of vector-valued functions

Let C be the curve traced out by $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ where f, g, h are differentiable functions on (a, b).

this is where t lives. It could be $(-\infty, \infty)$

Then $\vec{r}$ has a **derivative** (or is **differentiable**) on (a,b) and

$\vec{r}\,'(t) = \langle f'(t), g'(t), h'(t) \rangle$.

If $\vec{r}\,'(t) \neq \vec{0}$, then $\vec{r}\,'(t)$ is called the **tangent vector** at the point corresponding to t.