Math 2120

4/30/20

11.8 continued...

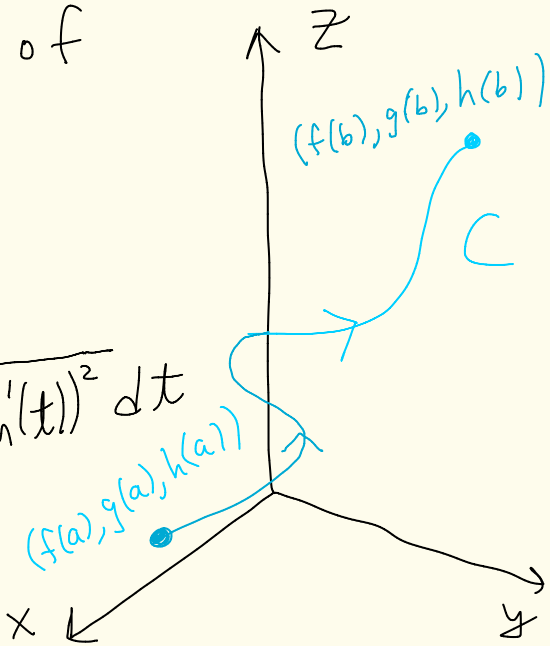
Arclength in 3D

Suppose a curve is given by $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ where $a \leq t \leq b$. Suppose $f'(t)$, $g'(t)$, and $h'(t)$ exist and are continuous for $a \leq t \leq b$.

The arclength of C is

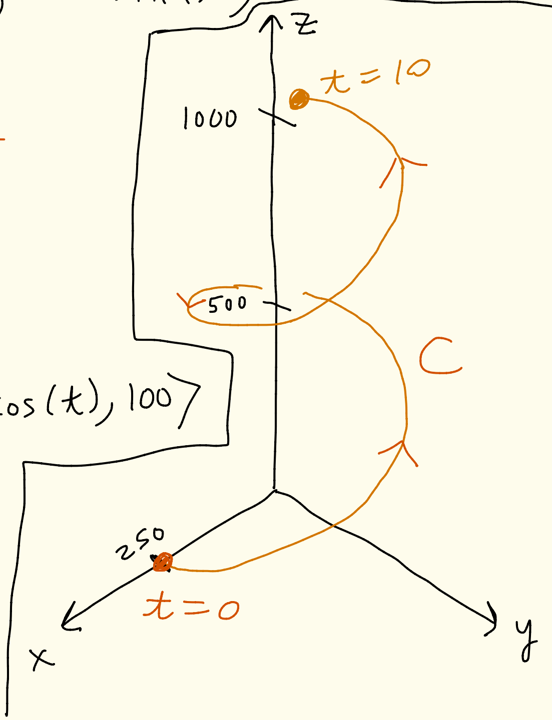
$$\int_a^b |\vec{r}\,'(t)|\, dt$$

$$= \int_a^b \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2}\, dt$$



Ex: Find the arclength given by $\vec{r}(t) = \langle 250\cos(t), 250\sin(t), 100t \rangle$ where $0 \le t \le 10$.

pg 2



$\vec{r}(0) = \langle 250, 0, 0 \rangle$

$\vec{r}(10) \approx \langle -210, -136, 1000 \rangle$

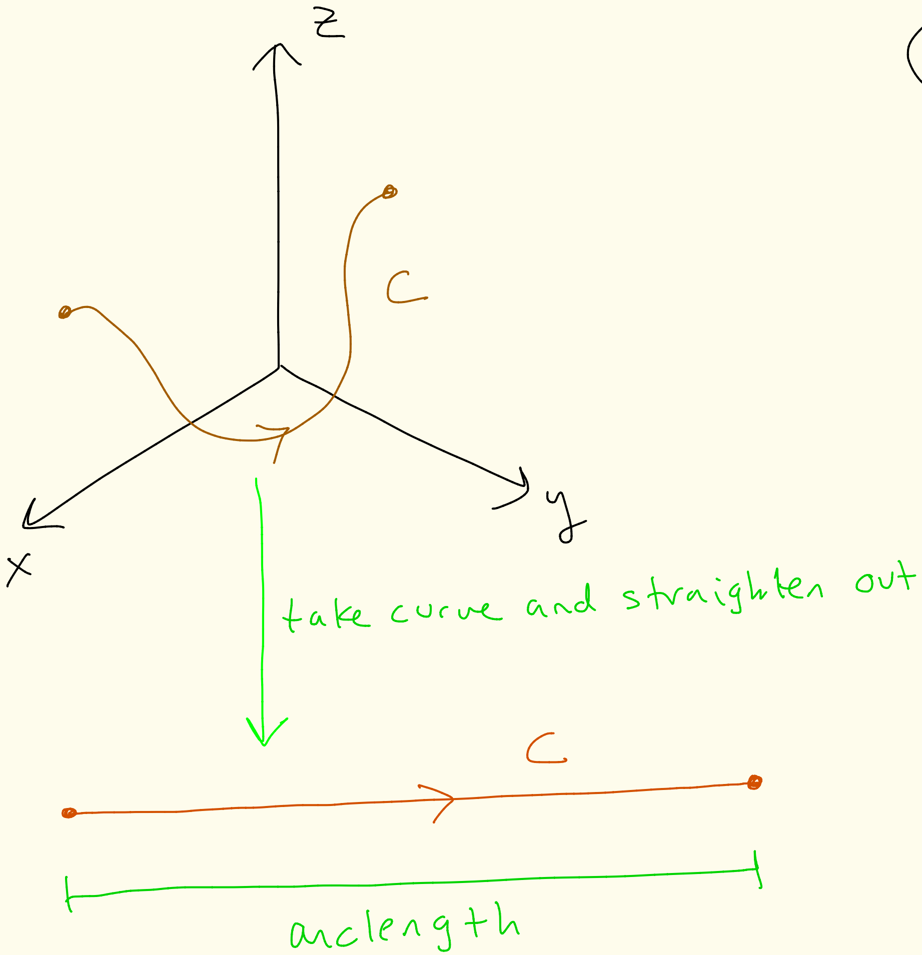
$\vec{r}\,'(t) = \langle -250\sin(t), 250\cos(t), 100 \rangle$

arclength =

$$= \int_0^{10} |\vec{r}\,'(t)|\, dt = \int_0^{10} \sqrt{(-250\sin(t))^2 + (250\cos(t))^2 + (100)^2}\, dt$$

$$= \int_0^{10} \sqrt{250^2 \underbrace{(\sin^2(t) + \cos^2(t))}_{1} + 100^2}\, dt$$

$$= \int_0^{10} \sqrt{72{,}500}\, dt = \sqrt{72500}\, t \Big|_0^{10} = 10\sqrt{72{,}500} \approx 2692.58$$

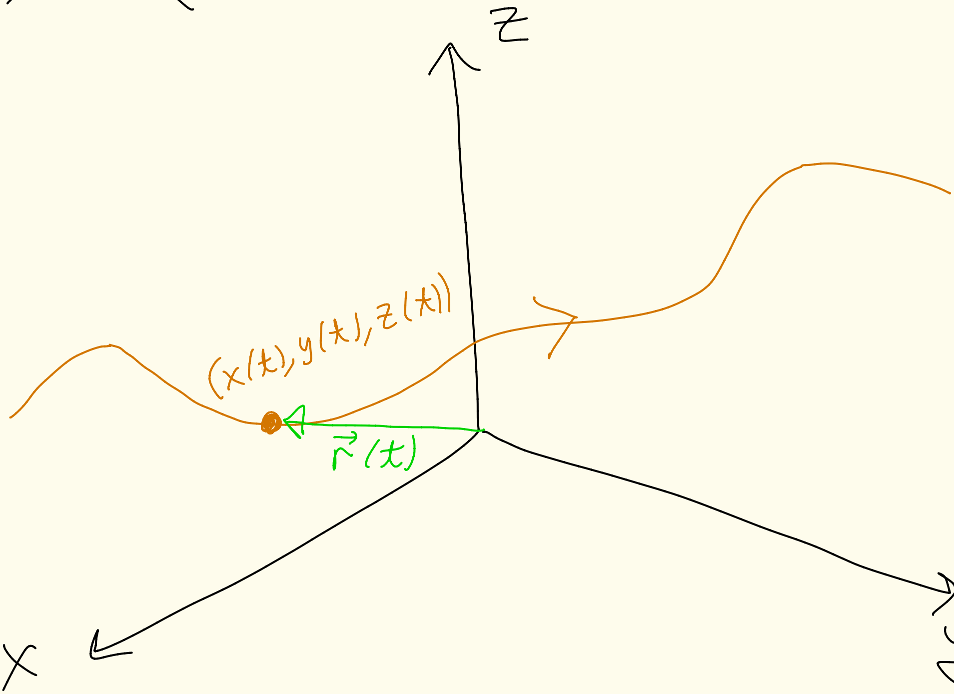

z
y
x
C
take curve and straighten out
C
arclength

11.7 - Motion in space

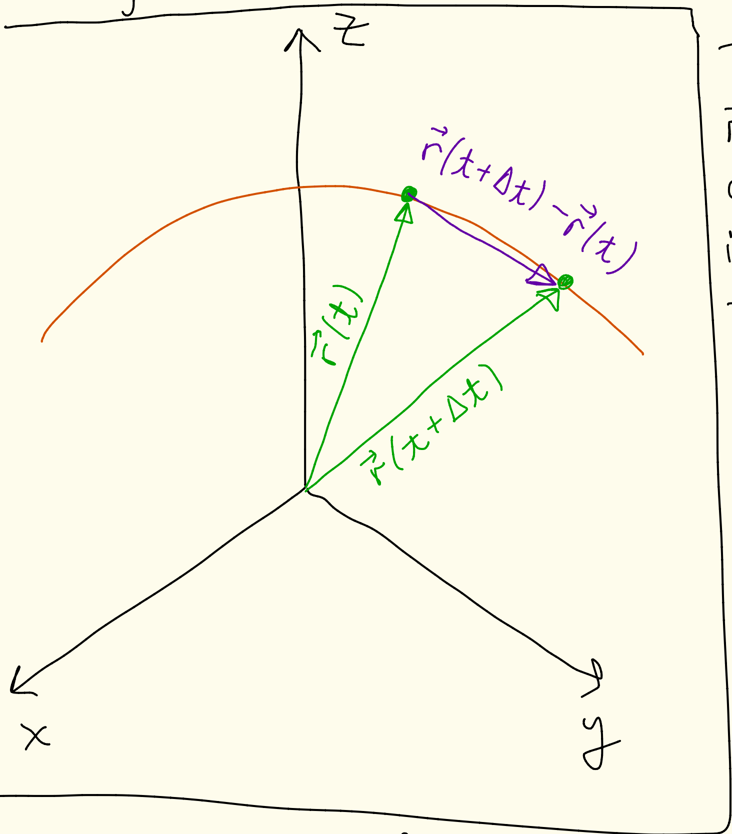
This section is not on the final.

Let the position of an object moving in three-dimensions be given by

$$\vec{r}(t) = \left\langle x(t), y(t), z(t) \right\rangle$$



Let's get a formula for the velocity at time t, that is the instantaneous change of position at time t.



The vector $\vec{r}(t+\Delta t)-\vec{r}(t)$ gives the change in position over the time period Δt.

What we want is

$$\frac{(\text{change in position})}{(\text{change in time})} = \frac{\vec{r}(t+\Delta t)-\vec{r}(t)}{\Delta t}$$

This number would give us the average velocity over time period t to $t+\Delta t$.

We want the instantaneous velocity at t so we take the limit as $\Delta t \to 0$.

$$\lim_{\Delta t \to 0} \frac{\vec{r}(t+\Delta t)-\vec{r}(t)}{\Delta t} =$$

pg 6

$$= \lim_{\Delta t \to 0} \frac{\overbrace{\langle x(t+\Delta t), y(t+\Delta t), z(t+\Delta t)\rangle}^{\vec{r}(t+\Delta t)} - \overbrace{\langle x(t), y(t), z(t)\rangle}^{\vec{r}(t)}}{\Delta t}$$

$$= \lim_{t \to 0} \frac{\langle x(t+\Delta t) - x(t),\ y(t+\Delta t) - y(t),\ z(t+\Delta t) - z(t)\rangle}{\Delta t}$$

$$= \lim_{t \to 0} \left\langle \frac{x(t+\Delta t) - x(t)}{\Delta t}, \frac{y(t+\Delta t) - y(t)}{\Delta t}, \frac{z(t+\Delta t) - z(t)}{\Delta t} \right\rangle$$

$$= \left\langle \lim_{t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}, \lim_{t \to 0} \frac{y(t+\Delta t) - y(t)}{\Delta t}, \lim_{t \to 0} \frac{z(t+\Delta t) - z(t)}{\Delta t} \right\rangle$$

this is a property of limits of vector functions

$$= \langle x'(t), y'(t), z'(t)\rangle = \vec{r}\,'(t)$$

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$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$\vec{r}(t) \leftarrow$ position

$\vec{r}\,'(t) \leftarrow$ velocity

$|\vec{r}\,'(t)| \leftarrow$ speed

$\vec{r}\,''(t) \leftarrow$ acceleration (change in velocity)