Math 2120

4/6/20

Monday

week 11

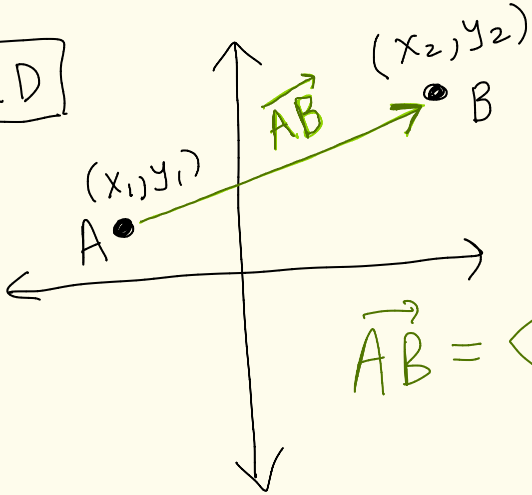
Tonight I'm going to email you a password <u>for zoom</u> that will go into effect starting tomorrow

It might just be a new zoom link with the password embedded into it.

11.1/11.2 continued...

Let A and B be two points.
Let's construct the vector $\overrightarrow{AB}$ from A to B.

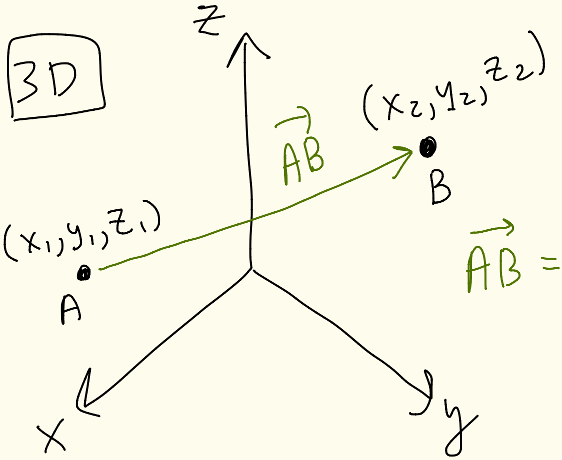
$\boxed{\text{2D}}$



$$\overrightarrow{AB} = \langle x_2 - x_1 \,,\, y_2 - y_1 \rangle$$

(endpoint − initial point)

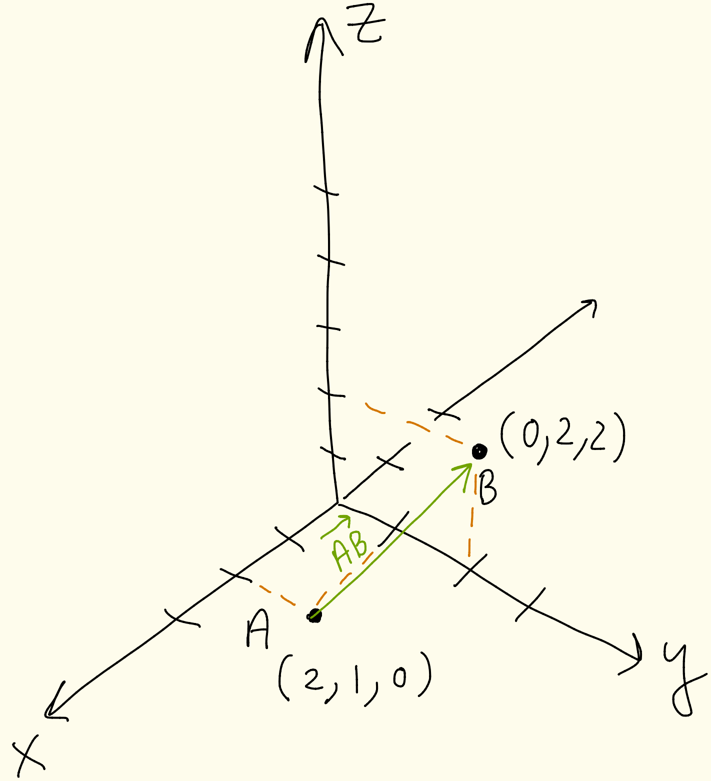
$\boxed{\text{3D}}$



$$\overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

(endpoint − initial point)

Ex: $A = (2,1,0)$
$B = (0,2,2)$



$$\vec{AB} = \langle 0-2,\ 2-1,\ 2-0 \rangle$$
$$= \langle -2,\ 1,\ 2 \rangle$$

Vector properties

If $\vec{a}$, $\vec{b}$, and $\vec{c}$ are vectors and α and β are scalars, then:

1. $\vec{a}+\vec{b} = \vec{b}+\vec{a}$
2. $\vec{a}+\vec{0} = \vec{0}+\vec{a} = \vec{a}$
3. $\alpha(\vec{a}+\vec{b}) = \alpha\vec{a}+\alpha\vec{b}$
4. $(\alpha\beta)\vec{a} = \alpha(\beta\vec{a})$
5. $\vec{a}+(\vec{b}+\vec{c})$
   $= (\vec{a}+\vec{b})+\vec{c}$
6. $\vec{a}+(-\vec{a}) = \vec{0}$
7. $(\alpha+\beta)\vec{a}$
   $= \alpha\vec{a}+\beta\vec{a}$

Greek letters
α alpha
β beta

2D

$\vec{0} = \langle 0, 0 \rangle$

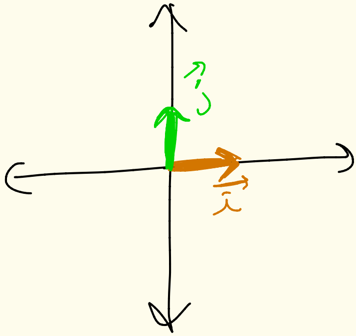
3D

$\vec{0} = \langle 0, 0, 0 \rangle$

2D	$\vec{a} = \langle x, y \rangle$ $-\vec{a} = \langle -x, -y \rangle$
3D	$\vec{a} = \langle x, y, z \rangle$ $-\vec{a} = \langle -x, -y, -z \rangle$

2D

$$\vec{i} = \langle 1, 0 \rangle$$
$$\vec{j} = \langle 0, 1 \rangle$$



Given any vector $\boxed{\vec{v} = \langle x, y \rangle}$, then

$$\begin{aligned}
\vec{v} = \langle x, y \rangle &= \langle x, 0 \rangle + \langle 0, y \rangle \\
&= x \langle 1, 0 \rangle + y \langle 0, 1 \rangle \\
&= x\,\vec{i} + y\,\vec{j}
\end{aligned}$$

$\vec{v}$ has two expressions

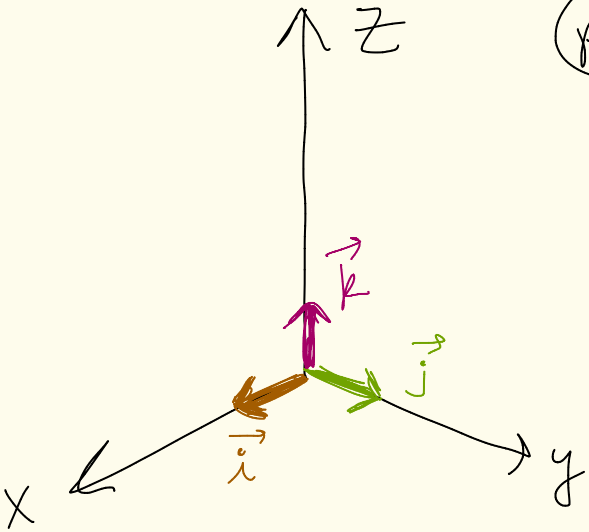
$$\boxed{\begin{aligned} \vec{v} &= \langle x, y \rangle \\ \vec{v} &= x\,\vec{i} + y\,\vec{j} \end{aligned}}$$

3D

$\vec{i} = \langle 1, 0, 0 \rangle$

$\vec{j} = \langle 0, 1, 0 \rangle$

$\vec{k} = \langle 0, 0, 1 \rangle$



$$\vec{v} = \langle x, y, z \rangle$$
$$= x\langle 1, 0, 0 \rangle + y\langle 0, 1, 0 \rangle + z\langle 0, 0, 1 \rangle$$
$$= x\vec{i} + y\vec{j} + z\vec{k}$$

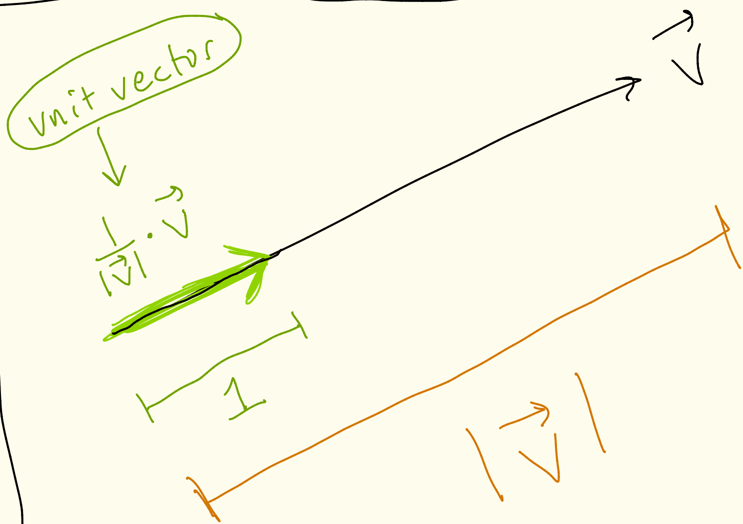
$\vec{v}$ has two expressions:

$$\vec{v} = \langle x, y, z \rangle$$
$$\vec{v} = x\vec{i} + y\vec{j} + z\vec{k}$$

A <u>unit vector</u> is a vector with magnitude/length equal to 1.

Given a vector $\vec{v} \neq \vec{0}$, then the unit vector in the direction of $\vec{v}$ is $\frac{1}{|\vec{v}|} \cdot \vec{v} = \frac{\vec{v}}{|\vec{v}|}$

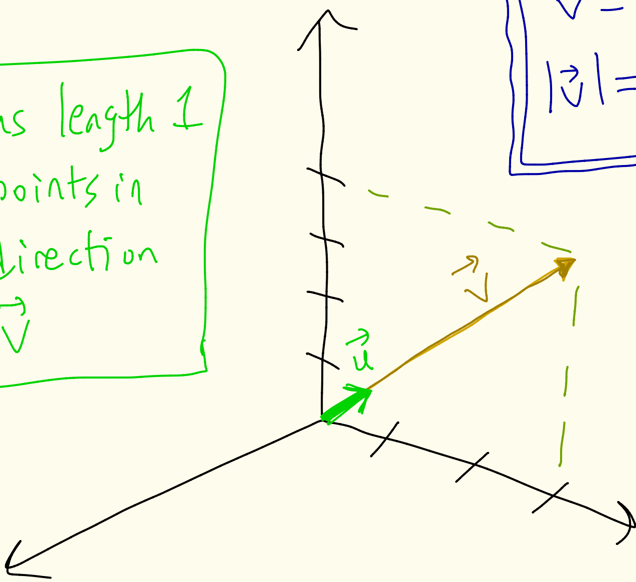
PICTURE



Ex: Find the unit vector $\vec{u}$ (pg 8) in the direction of $\vec{v} = \langle 0, 3, 4 \rangle$

$$\boxed{\begin{aligned} \vec{v} &= \langle x, y, z \rangle \\ |\vec{v}| &= \sqrt{x^2 + y^2 + z^2} \end{aligned}}$$

u has length 1 and points in the direction of $\vec{v}$



$$\vec{u} = \frac{1}{|\vec{v}|} \vec{v} = \frac{1}{\sqrt{0^2 + 3^2 + 4^2}} \langle 0, 3, 4 \rangle$$

$$\vec{u} = \frac{1}{5} \langle 0, 3, 4 \rangle$$

$$\vec{u} = \left\langle 0, \frac{3}{5}, \frac{4}{5} \right\rangle$$

Workshop

8.5 Do these converge or diverge?

(28) $$\sum_{k=1}^{\infty} \frac{k^2+k-1}{k^4+4k^2-3}$$

$$\frac{k^2+k-1}{k^4+4k^2-3} \approx \frac{k^2}{k^4} = \frac{1}{k^2}$$

large k

Compare with $\sum_{k=1}^{\infty} \frac{1}{k^2}$

converges
$p=2$ series

$$L = \lim_{k\to\infty} \frac{\left(\frac{k^2+k-1}{k^4+4k^2-3}\right)}{\left(\frac{1}{k^2}\right)} = \lim_{k\to\infty} k^2\left(\frac{k^2+k-1}{k^4+4k^2-3}\right)$$

$$= \lim_{k\to\infty} \frac{k^4-k^3-k^2}{k^4+4k^2-3} = \lim_{k\to\infty} \frac{1-\frac{1}{k}-\frac{1}{k^2}}{1+\frac{4}{k^2}-\frac{3}{k^4}}$$

divide by k^4
top / bottom

$$= \frac{1-0-0}{1+0-0} = 1 = L$$

Since $0 < L < \infty$, and $\sum \frac{1}{k^2}$ converges, we know $\sum \frac{k^2+k-1}{k^4+4k^2-3}$ converges

$\boxed{8.5}$ pg 10

16 $\displaystyle\sum_{k=1}^{\infty} \frac{k^6}{k!}$ Converge or diverge?

Ratio test

$$L = \lim_{k\to\infty} \frac{\left|\dfrac{(k+1)^6}{(k+1)!}\right|}{\left|\dfrac{k^6}{k!}\right|} = \lim_{k\to\infty} \frac{k!}{k^6}\cdot\frac{(k+1)^6}{(k+1)!}$$

$$= \lim_{k\to\infty} \frac{\cancel{k!}}{k^6}\cdot\frac{(k+1)^6}{(k+1)\left[\cancel{k!}\right]}$$

$$= \lim_{k\to\infty} \frac{(k+1)^6}{k^6(k+1)} = \lim_{k\to\infty} \frac{(k+1)^5}{k^6}$$

$$= \lim_{k\to\infty} \frac{k^5+5k^4+10k^3+10k^2+5k+1}{k^6}$$

$$\begin{array}{c} 1 \\ 1\ \ 1 \\ 1\ \ 2\ \ 1 \\ 1\ \ 3\ \ 3\ \ 1 \\ 1\ \ 4\ \ 6\ \ 4\ \ 1 \\ 1\ \ 5\ \ 10\ \ 10\ \ 5\ \ 1 \end{array}$$

$$= \lim_{k\to\infty} \frac{1}{k}+\frac{5}{k^2}+\frac{10}{k^3}+\frac{10}{k^4}+\frac{5}{k^5}+\frac{1}{k^6}$$

$$= 0+0+0+0+0+0 = 0 = L.$$

Since $0 \le L < 1$ the series converges.

Could also use L'Hospital multiple times

$$\lim_{k\to\infty} \frac{(k+1)^5}{k^6} \stackrel{L'H}{=} \lim_{k\to\infty} \frac{5(k+1)^4}{6k^5} \stackrel{L'H}{=} \cdots$$

$$\cdots \stackrel{L'H}{=} \lim_{k\to\infty} \frac{5\cdot4\cdot3\cdot2}{6\cdot5\cdot4\cdot3\cdot2\cdot k} = 0$$

<u>Ex:</u> $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2+4}$

$$\frac{1}{k^2+4} \leq \frac{1}{k^2}$$

$\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$ converges, so $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2+4}$ converges

---

<u>Ex:</u> $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2-4}$ $\qquad \displaystyle\frac{1}{k^2-4} \not\leq \frac{1}{k^2}$

Use limit comp test instead.

pg 12

$$\sum_{k=1}^{\infty} \frac{\ln(k)}{k}$$

Note: $\frac{\ln(k)}{k} \geq \frac{1}{k}$

$\sum_{k=1}^{\infty} \frac{1}{k}$ diverges

So,

$\sum_{k=1}^{\infty} \frac{\ln(k)}{k}$ diverges by comparison test

Test

1. posted at 11am Weds on course website & email.
2. You pick a 2 hr window to work on test in this 24 hour window (Weds 11am – Thurs 11am)

2 hr consecutive window such as 12pm – 2pm

No breaking 2 hrs into chunks! (ie. no 12-1 & 4-5)

3. scan & email back to me by Thursday at 11am.

(4) Weds — NO CLASS OR WORKSHOP

(5) Put problems in order.

(6) There will be instructions on a statement to write & sign