Math 2120

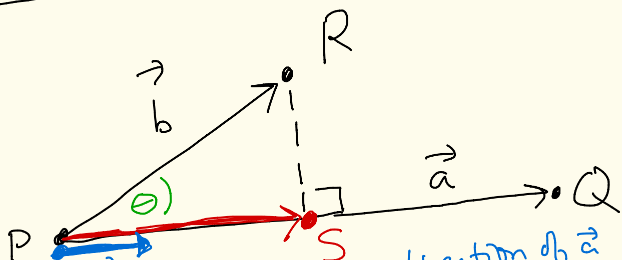
4/9/20

1. I'll grade the tests by this weekend and email your test to you.

2. The math dept might be standardizing our final across the sections. If so, they are working on a set do study problems.

continued from last time

we want to "project" $\vec{b}$ onto $\vec{a}$. We had this picture last time



$\frac{\vec{a}}{|\vec{a}|}$ = unit vector in direction of $\vec{a}$

$|\vec{PS}| = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$

$\vec{PS} = \text{proj}_{\vec{a}}(\vec{b})$.

Recall

$\vec{v}\cdot\vec{w} = |\vec{v}||\vec{w}|\cos(\theta)$

length of $\vec{PS}$: $\cos(\theta) = \frac{|\vec{PS}|}{|\vec{b}|}$

So, $|\vec{PS}| = |\vec{b}|\cos(\theta) = |\vec{b}|\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$= \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$

To make $\vec{PS}$, multiply a unit vector (length 1) in the direction of $\vec{a}$ by $|\vec{PS}|$. So, $\text{proj}_{\vec{a}}(\vec{b}) = \vec{PS} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\cdot\frac{\vec{a}}{|\vec{a}|}$.

$$\text{proj}_{\vec{a}}(\vec{b}) = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\cdot\frac{\vec{a}}{|\vec{a}|}$$

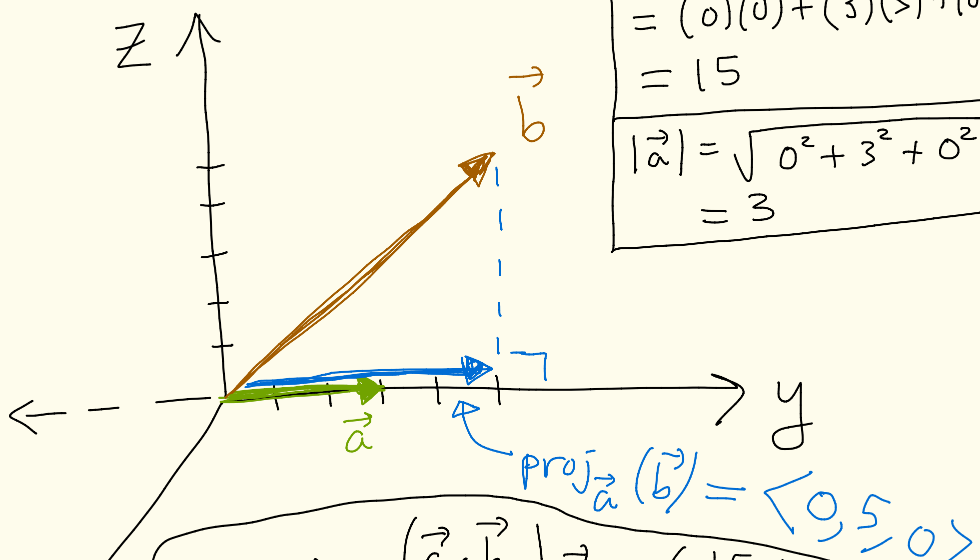
$$= \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\vec{a}$$

$\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2} \leftarrow$ number

Ex: Project $\vec{b} = \langle 0,5,5\rangle$ onto $\vec{a} = \langle 0,3,0\rangle$.

$$\vec{a}\cdot\vec{b} = (0)(0)+(3)(5)+(0)(5) = 15$$

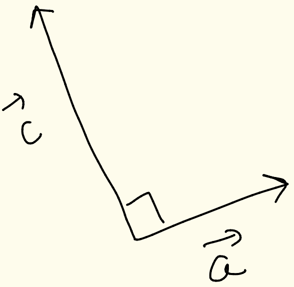
$$|\vec{a}| = \sqrt{0^2+3^2+0^2} = 3$$

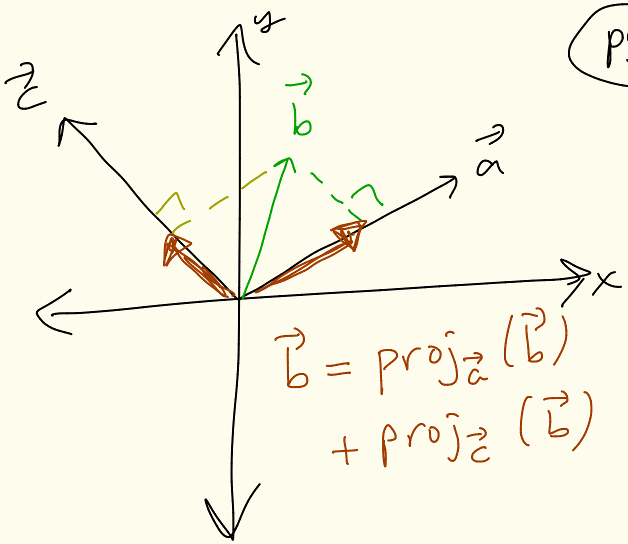


$$\text{proj}_{\vec{a}}(\vec{b}) = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\vec{a} = \left(\frac{15}{3^2}\right)\langle 0,3,0\rangle$$

$$= \left\langle \tfrac{15}{9}\cdot 0,\ \tfrac{15}{9}\cdot 3,\ \tfrac{15}{9}\cdot 0\right\rangle = \langle 0,5,0\rangle$$

Class question





$$\vec{b} = \text{proj}_{\vec{a}}(\vec{b}) + \text{proj}_{\vec{c}}(\vec{b})$$

11.4 – Cross products

Def: A determinant of order 2 is

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \qquad \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Ex: $\begin{vmatrix} 2 & 1 \\ 5 & 3 \end{vmatrix} = 2 \cdot 3 - 1 \cdot 5$

$= 1$

Def: The cross product of $\vec{v} = \langle a, b, c \rangle$ and $\vec{w} = \langle d, e, f \rangle$ is

$$\vec{v} \times \vec{w} = \underbrace{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ d & e & f \end{vmatrix}}_{3\times 3 \text{ determinant}}$$

$$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$$

$$= \vec{i}\underbrace{\begin{vmatrix} b & c \\ e & f \end{vmatrix}}_{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ d & e & f \end{vmatrix}} - \vec{j}\underbrace{\begin{vmatrix} a & c \\ d & f \end{vmatrix}}_{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ d & e & f \end{vmatrix}} + \vec{k}\underbrace{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}_{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ d & e & f \end{vmatrix}}$$