Math 2120

5/4/20

- Test 3: I'll email everyone today their tests hopefully

10.1

(15) Eliminate the variable to obtain a formula for x, y.

$$x = \sqrt{t} + 4 \qquad 0 \le t \le 16$$
$$y = 3\sqrt{t}$$

$$\sqrt{t} = x - 4$$
$$y = 3\sqrt{t} = 3(x-4) = 3x - 12$$

$$y = 3x - 12$$

Range for x

$0 \le t \le 16$
$0 \le \sqrt{t} \le 4$
$0 \le x - 4 \le 4$
$4 \le x \le 8$

Answer:

$y = 3x - 12$
$4 \le x \le 8$

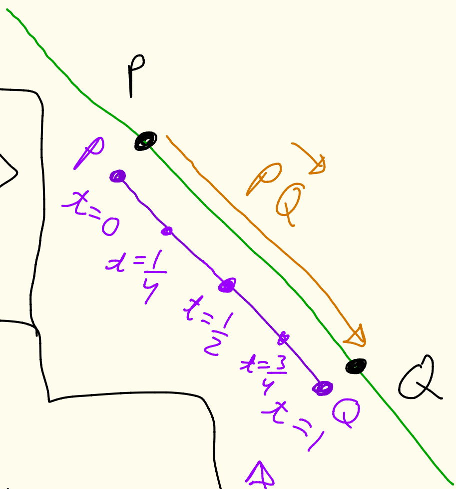
10.1

pg 2

43 Find parametric equations for the line segment from $P(-1,-3)$ to $Q(6,-16)$

We need a vector that is in the direction of the line.

$$\overrightarrow{PQ} = \langle 6-(-1), -16-(-3) \rangle$$
$$= \langle 7, -13 \rangle$$



line through P and Q

$$x = -1 + 7t$$
$$y = -3 - 13t$$

t any real #

$P(-1,3) + t\overrightarrow{PQ}$

(purple part of line)

$$\boxed{\begin{aligned} x &= -1 + 7t \\ y &= -3 - 13t \\ 0 &\le t \le 1 \end{aligned}}$$

To get just the segment $\overline{PQ}$ you let t be between 0 & 1

$\boxed{10.3}$

$\theta = \frac{\pi}{12}$ | $r = \cos\left(\frac{\pi}{4}\right) \approx 0.7$

Find the area of

(27) The region inside one leaf of $r = \cos(3\theta)$.

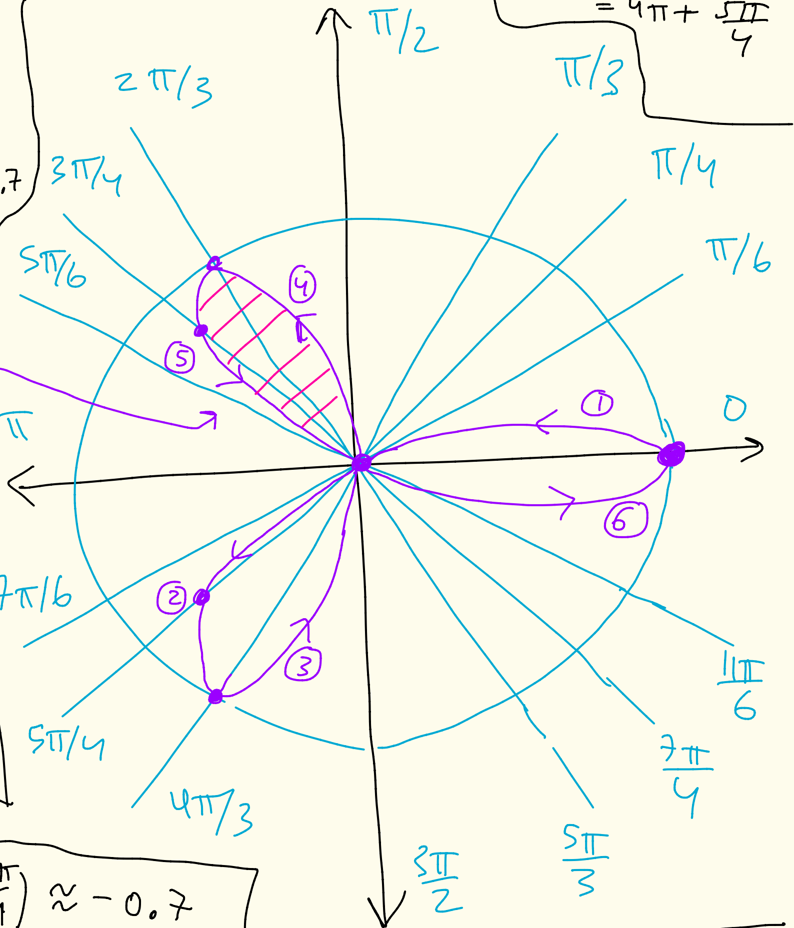
$$\frac{21\pi}{6} = \frac{12\pi}{6} + \frac{9\pi}{6} = 2\pi + 3\pi/2$$

$$\frac{15\pi}{4} = \frac{8\pi}{4} + \frac{7\pi}{4} = 2\pi + \frac{7\pi}{4}$$

pg 3

$$\frac{21\pi}{4} = \frac{16\pi}{4} + \frac{5\pi}{4} = 4\pi + \frac{5\pi}{4}$$

θ	$r = \cos(3\theta)$
0	1
$\pi/6$	$\cos\left(\frac{\pi}{2}\right) = 0$
$\pi/4$	$\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2} \approx -0.7$
$\pi/3$	-1
$\pi/2$	0
$\frac{2\pi}{3}$	1
$\frac{3\pi}{4}$	$\cos\left(\frac{9\pi}{4}\right) = \frac{\sqrt{2}}{2} \approx 0.7$
$\frac{5\pi}{6}$	0
π	-1
$\frac{7\pi}{6}$	$\cos\left(\frac{21\pi}{6}\right) = \cos\left(\frac{3\pi}{2}\right) = 0$
$5\pi/4$	$\cos\left(\frac{15\pi}{4}\right) = \cos\left(\frac{7\pi}{4}\right) \approx 0.7$
$4\pi/3$	1
$3\pi/2$	0
$5\pi/3$	-1
$7\pi/4$	$\cos\left(\frac{21\pi}{4}\right) = \cos\left(\frac{5\pi}{4}\right) \approx -0.7$
$\frac{11\pi}{6}$	$\cos\left(\frac{33\pi}{6}\right) = \cos\left(\frac{3\pi}{2}\right) = 0$



$$\frac{6\pi}{4} = \frac{3\pi}{2}$$

$$\frac{33\pi}{6} = \frac{24\pi}{6} + \frac{9\pi}{6} = 4\pi + \frac{3\pi}{2}$$

$$\cos(2\pi + \theta) = \cos(\theta)$$

(p94)

$$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$$

Area of one leaf:

$$\int_{\pi/2}^{5\pi/6} \frac{1}{2}\left(\cos(3\theta)\right)^2 d\theta$$

$$= \frac{1}{2}\int_{\pi/2}^{5\pi/6} \cos^2(3\theta)\, d\theta = \frac{1}{2}\int_{\pi/2}^{5\pi/6} \frac{1}{2} + \frac{1}{2}\cos(6\theta)\, d\theta$$

$$= \frac{1}{4}\int_{\pi/2}^{5\pi/6} \left(1 + \cos(6\theta)\right) d\theta$$

$$= \frac{1}{4}\left[\theta + \frac{1}{6}\sin(6\theta)\right]_{\pi/2}^{5\pi/6}$$

$$= \frac{1}{4}\left[\left(\frac{5\pi}{6} + \frac{1}{6}\underbrace{\sin\left(6\cdot\frac{5\pi}{6}\right)}_{\sin(5\pi)=0}\right) - \left(\frac{\pi}{2} + \frac{1}{6}\underbrace{\sin\left(6\cdot\frac{\pi}{2}\right)}_{\sin(3\pi)=0}\right)\right]$$

$$= \frac{1}{4}\left[\frac{5\pi}{6} - \underbrace{\frac{\pi}{2}}_{\boxed{3\pi/6}}\right] = \frac{1}{4}\left[\frac{2\pi}{6}\right] = \frac{\pi}{12} \approx 0.2617$$