Math 2120 - Spring 2020 - Test 3

Directions:

(a) Pick a consecutive 2-hour window to take this exam, such as 12pm - 2pm. You may only use 2 hours of consecutive time. Do not split the time (like 12-1pm and then 5-6pm).

(b) You can only use your mind to take this exam. No help from any sources or people. No books, no notes, no online, etc.

(c) No calculators.

(d) Use blank paper (like printer paper) if you have it please.

(e) On the first page of your exam, before any of your solutions, put these three things:

  (a) Your name.

  (b) The time period that you chose. (Such as 12pm - 2pm on Weds)

  (c) Copy this statement and then sign your signature after it:

  "Everything on this test is my own work. I did not use any sources or talk to anyone about this exam." your signature

(f) After your name and the above statement with signature, start putting your solutions to the problems. Please put them in order. That is, first problem 1, then problem 2, etc. You can put each one on its own page.

(g) Scan and email to me by Thursday the 30th at 11am.

The problems are on the next page.

Recall: When you find the interval of convergence of a power series make sure to test the endpoints. When you check the endpoints you can just say converges / diverges by such and such test without going through the details of the test. When you find the radius of converge you don't have to check the endpoints.

1. [10 points] Find the interval of convergence of the following power series:

$$\sum_{k=1}^{\infty}(-1)^k\frac{(x-4)^k}{2^k}$$

2. [10 points] Find a power series representation centered around 0 for the following function.

$$f(x)=\frac{-3x^2}{(1+x^3)^2}$$

What is the radius of convergence?

3. [10 points] Integrate/differentiate a known power series or use the formula for Maclaurin series to find the Maclaurin series for

$$g(x)=\ln(1+x)$$

What is the interval of convergence?

4. [10 points] Evaluate the following limit using power series.

$$\lim_{x\to 0}\frac{\sin(x)}{x}$$

5. [20 points - 5 each] Consider the points $P(1,0,1)$ and $Q(2,-1,3)$. Let $\vec{u}=\langle 3,0,4\rangle$ and $\vec{v}=\left\langle -3,2,\frac{9}{4}\right\rangle$ and $\vec{w}=\langle 1,2,2\rangle$

(a) Find $\overrightarrow{PQ}$

(b) Find a unit vector in the direction of $\vec{u}$

(c) Are $\vec{u}$ and $\vec{v}$ orthogonal? Why or why not?

(d) Compute the projection of $\vec{u}$ onto $\vec{w}$.

6. [10 points] Let $\vec{v}=\langle -1,0,6\rangle$ and $\vec{w}=\langle 2,-5,-3\rangle$. Calculate $\vec{v}\times\vec{w}$.