Math 5402 - Spring 2020 - Test 2

Directions:

(a) Pick a consecutive 2-hour window to take this exam, such as 12pm-2pm or 3:45pm-5:45pm. You may only use 2 hours of consecutive time. Do not split the time (like 12-1pm and then 5-6pm).

(b) You can only use your mind to take this exam. No help from any sources or people. No books, no notes, no online, etc.

(c) No calculators.

(d) Use blank paper (like printer paper) if you have it please.

(e) On the first page of your exam, before any of your solutions, put these three things:

  (a) Your name.

  (b) The time period that you chose. (Such as 3:45pm-5:45pm on Weds)

  (c) Copy this statement and then sign your signature after it:

  "Everything on this test is my own work. I did not use any sources or talk to anyone about this exam." **your signature** ______

(f) After your name and the above statement with signature, start putting your solutions to the problems. Please put them in order. That is, first problem 1, then problem 2, etc. You can put each one on its own page.

(g) Scan and email to me by Thursday the 23rd at 7pm.

The problems are on the next page.

DO ALL THREE OF PROBLEMS #1, # 2, and # 3.

1. Consider the ring $K = \mathbb{Q}[x]/(x^3 - 5)$.

(a) Is K an integral domain? Is K a field? Justify your answers.

(b) Use set theory notation to give a description of all of the elements of K.

2.

(a) Construct a field $\mathbb{F}_{25}$ of size 25. Go through the details of how you obtained the construction and why it results in a field. After describing the construction, give a description of all of the elements in the field $\mathbb{F}_{25}$ (you can either list all 25 of them or write the description in set theory notation).

(b) Pick an element from $\mathbb{F}_{25}$ that is not in $\mathbb{Z}_5$ and take the 3-rd power of your chosen element. Reduce your answer so it conforms to your description from part (a).

3. Let $K = \mathbb{Q}(\alpha)$ where $\alpha = \sqrt{3 + \sqrt{6}}$. Determine $[K : \mathbb{Q}]$ and use set theory notation to give a description of all of the elements of K. Justify your answers.

PICK TWO PROOFS FROM BELOW.

A. Let R be an integral domain. Let u be a unit of R and let x be an irreducible element of R. Prove that ux is an irreducible element of R.

B. Let K be a field extension of a field F. Prove that $K = F$ if and only if $[K : F] = 1$.

C. Let K be a field extension of a field F with $[K : F] = n$. Let $f(x) \in F[x]$ be of degree $m > 1$ such that $f(x)$ is irreducible over F.

(a) Prove that if $\gcd(m, n) = 1$ then f has no root in K.

(b) Is the converse true? Prove or give a counter-example. The converse is: If f has no root in K, then $\gcd(m, n) = 1$.