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A Guide for the Perplexed: What Mathematicians Need to Know to Understand Philosophers of Mathematics

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This note is a guide for mathematicians who don't know much about the philosophy of mathematics—a guide that explains how to read philosophers of mathematics. I hope to make clear for mathematicians what philosophers of mathematics are really up to and eliminate some confusions.

The picture I provide here is controversial. This is par for the course in philosophy—we philosophers disagree about almost everything. I will try to indicate *where* it's controversial, and as will become clear, some of what I say can be seen as a partial justification of my position.

Second, I present a picture of only a *part* of the philosophy of mathematics.

Clearing Up Some Confusions About the Philosophy of Mathematics

The two main confusions about the philosophy of mathematics I will try to clear up concern the following questions:

1. What is the relationship between mathematics and the philosophy of mathematics?
2. What kinds of theories are philosophers of mathematics putting forward?

These two questions are deeply related. Confusion about question number 2 leads to confusion about question number 1. So let me start with question number 2. What kinds of theories are philosophers of mathematics putting forward?

You may think that the philosophy of mathematics is an eternal debate between those who argue that (a) abstract mathematical objects exist in a nonphysical, nonmental, nonspatiotemporal platonic realm, and (b) mathematics is a mental or social construction. But this isn't the only thing that's going on, and what's more, the arguments for these theories are ultimately driven by theories of an entirely different kind.

First, let's say that an *abstract object*, or a *platonic object*, is a nonphysical, nonmental, nonspatiotemporal object. *Platonism* is the view that there really are such things, and *antiplatonism* is the view that there aren't. There are various kinds of objects that platonists think are abstract objects, but the only ones that will matter here are mathematical objects—things such as numbers, and sets, and functions. Platonism obviously goes back to Plato (see, e.g., the *Meno* and the *Phaedo*), but numerous people have also endorsed it since then, including Frege (1884), Russell (1912), and Gödel (1964).

Second, I want to introduce the notion of an *ontological theory*. We can say that *ontology* is the branch of rational inquiry that's concerned with cataloguing the various kinds of objects that exist. A specific ontological theory is a theory about what sorts of things really exist. For instance, the claim that there are mermaids is a false ontological theory, and the claim that there are Tasmanian devils is a true ontological

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theory. Platonism, then, as I defined it previously, is an ontological theory.

Now, you might think that philosophers of mathematics spend their time arguing about the truth of a certain ontological theory, namely, platonism. But this is an oversimplification, in two different ways. First, philosophers of mathematics are concerned with other questions. And, second, even when ontology is the *ultimate* concern, it is often in the background. Ontological theories such as platonism and antiplatonism are often the ultimate *conclusions* of philosophical arguments, but they are best thought of as following from theories of a completely different kind. This other kind of theory is a *semantic* theory; I'll say in a moment what a semantic theory is.

First let me acknowledge briefly that there is more to the philosophy of mathematics than ontology. For instance, philosophers are also interested in questions about the *applications* of mathematics (i.e., the use that's made of mathematics in empirical science) and the *epistemology* of mathematics (i.e., the nature of mathematical *knowledge*). But there is still a crucial link here to ontology. For instance, a famous objection to platonism (see, e.g., Benacerraf 1973) is that if our mathematical theories were really about nonspatiotemporal abstract objects, then mathematical knowledge would be impossible, because we humans don't have any way of acquiring information about such objects. I think it's fair to say that most of the work that's been done on the epistemology of mathematics has ultimately been concerned with supporting or responding to this objection to the ontological theory of platonism. Likewise, one of the most important objections to antiplatonist (in particular, antirealist) philosophies of mathematics is that they can't accommodate the usefulness of mathematics; and I think it's fair to say that most of the philosophical work that's been done on the applications of mathematics (see, e.g., Field 1980) has ultimately been concerned with supporting or responding to this objection to the ontological theory of antiplatonism.

Philosophers are also very interested in the legitimacy of things like computer proofs and experimental mathematics. These have nothing much to do with ontological questions about platonism and antiplatonism.

A semantic theory is a theory about what certain expressions mean (or refer to) in a specific language. So, for instance, the claim that the term "Mars" refers (in English) to the Empire State Building is a false semantic theory, and the claim that "Mars" refers (in English) to the fourth planet from the sun is a true semantic theory. (Actually, rather than saying that a semantic theory tells us what a word refers to in a certain language, it's better to say that a semantic theory tells us what a word is *supposed* to refer to, or what it *purports* to refer to, in a certain language. This allows us to adopt a semantic theory that tells us that, e.g., the term "Santa Claus" purports to refer to a jolly gift-giving man who wears a red suit and lives at the North Pole, without committing us to the claim that there really *is* such a creature.)

If the language in question is a *natural* language—if it's a language that's actually spoken by real people—then semantic theories of that language will be *empirical* theories.

When philosophers of mathematics argue for ontological theories like platonism and antiplatonism, their arguments are often primarily driven by semantic theories—in particular, by semantic theories of the language of ordinary mathematical discourse (or as philosophers sometimes call it, *mathematese*). To put the point in a somewhat exaggerated (but I think illuminating) way, we can say this: Philosophers of mathematics are centrally concerned with developing levelheaded theories of the semantics of ordinary mathematical discourse, and then they often use these theories to motivate bizarre ontological theories.

We now have an answer to question number 2: the central thing that philosophers of mathematics are doing—the thing that drives their ontological theories and their epistemological theories and so on—is developing semantic theories of the language of mathematics.

Moreover, this brings with it an answer to question number 1—i.e., the question about the relation between mathematicians and philosophers of mathematics. I see it as analogous to the relationship between a native speaker of French and a certain sort of *linguist*—a grammarian of French whose native tongue is English but who has learned a good deal of French in order to construct a grammar for that language. There is an obvious sense in which the native speaker of French knows her language better—indeed, *much* better—than the linguist does. But the linguist has been trained to construct syntactic theories, and most native speakers of French have not. Thus, while the linguist has to respect the linguistic intuitions of native speakers, he cannot very well ask them what the right theory is. Likewise, while it is obvious that mathematicians know mathematics (and the language of mathematics) better than philosophers do—indeed, *much* better—most of them have not been trained to construct semantic theories in the way that philosophers have. So while philosophers of mathematics have to respect the intuitions of mathematicians, they can't just ask them what the right theory is.

Many philosophers of mathematics would resist the analogy to linguists. In their view, their primary concern is with ontology and not semantics, because their ultimate goal is to uncover the metaphysical nature of reality. But when philosophers say they're not centrally concerned with semantics, they are often unaware of the degree to which their arguments depend on semantic theories. To justify this claim fully would take quite a bit of space, and I can't do the

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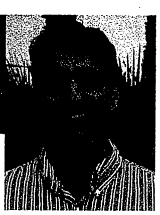

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whole job here. But let me make two points. First, in the remainder of this essay I take a traditional philosophical argument for an ontological theory and explain how to read it as being largely about semantics. Second, people can be mistaken about what their own work is about. For example, platonists say that the work of mathematicians is about abstract objects *even if some mathematicians don't realize this.* Analogously, I claim that the philosophy of mathematics is largely about semantics *even if some philosophers of mathematics don't realize this.*

Doing Some Empirical Semantics

To provide an example of what I've been talking about, I will construct an entirely empirical argument for a specific theory of the semantics of mathematese, i.e., for the language of mathematics. The semantic theory that I will be arguing for can be put like this:

> *Semantic Platonism*: Ordinary mathematical sentences like "2 + 2 = 4" and "3 is prime" are straightforward claims about abstract objects (or at any rate, they *purport* to be about abstract objects).

This is not an ontological theory, and it doesn't imply any ontological theories. In particular, it doesn't imply that platonism is true. This is extremely important, and it's worth pausing to make sure that the point is clear. Let me do this by switching to a different example. Suppose that a team of Martian linguists landed on Earth and started trying to construct semantic theories for our languages. Suppose in particular that they happened upon a Christian community that kept using the term "God." Next, suppose that one of the Martians proposed the hypothesis that they are using the term "God" as a nickname for Gödel. And finally, suppose that another of the Martians disagreed with this theory and proposed the following alternative:

> *Semantic Theism*: The term "God" refers (in English) to an omniscient, omnipotent, benevolent Being who created the world (or at any rate, the term "God" *purports* to refer to such a Being).

The Martian who put this theory forward might not himself believe in God. His theory is a theory about how Christians use a certain term. Thus, semantic theism does not imply theism. Likewise, semantic platonism does not imply platonism; in other words, you can endorse semantic platonism without believing in abstract objects.

Now I want to construct a straightforwardly empirical argument for this theory. The first premise of the argument is as follows:

> (1) Ordinary mathematical sentences such as "2 + 2 = 4" and "3 is prime" should be interpreted at face value; i.e., they should be interpreted *literally*. For instance, "3 is prime" should be interpreted as having the following logical form: *Object O has property P*. Thus, what "3 is prime" says is that a certain object (namely, the number 3) has a certain property (namely, the property of being prime).

This premise is extremely plausible, but let me say a bit to explain it and justify it. Consider the following sentences:

(M) Mars is round.
(O) Obama is a politician.
(E) The Eiffel Tower is made of metal.

All three of these sentences have the same logical form; they all say that a certain object has a certain property. In other words, they all have the form *Object O has property P*. Now, on the surface, it *seems* that "3 is prime" has this form as well; it seems to say that a certain object (namely, 3) has a certain property (namely, primeness). But we have to be careful here. For, sometimes, when a sentence seems on the surface to have one logical form, it really has a different logical form. Here's an example:

(A) The average accountant has 2.4 children.

The surface form of this sentence is similar to the sentences mentioned previously; it seems to be saying that a certain object (namely, the average accountant) has a certain property (namely, the property of having 2.4 children). But, of course, this isn't really what this sentence says. The deep logical form of the sentence is as follows: *On average, accountants have 2.4 children.*

Now one might try to argue that while "3 is prime" seems on the surface to say that a certain object has a certain property, that's not the deep logical form of the sentence. But to motivate a non-face-value interpretation for a given sentence, we have to motivate the claim that the speaker or speakers in question have a positive intention to be saying something other than what the sentence says literally. And there has to be empirical evidence for this claim. In the case of (A), there is a mountain of evidence that when ordinary people utter sentences like this, they don't mean to be saying what the sentence says on the surface—they actively intend to be saying something else—and so they should not be interpreted as speaking literally. But in the case of ordinary mathematical sentences such as "3 is prime," there is no evidence that people mean to be speaking nonliterally, or metaphorically, and so we should interpret them as speaking literally.

So that is the argument for premise (1). The second premise in the argument can be put like this:

> (2) Given that ordinary mathematical sentences like "2 + 2 = 4" and "3 is prime" should be interpreted at face value—i.e., as making straightforward claims about certain objects (namely, numbers) —we can't interpret them as being about physical or mental objects, and so we have to interpret them as being about abstract objects (or more precisely, we have to interpret them as *purporting* to be about abstract objects).

Now, let me remind you of two points I've already made: first, premise (2) should *not* be taken as implying that platonism is true (i.e., that there really are abstract objects); and second, premise (2) is an *empirical* claim. The idea here is that this is the best way to interpret the ordinary mathematical assertions of ordinary folk and ordinary mathematicians.

Note, however, that the advocate of (2) does not have to say that ordinary people *consciously intend* to be talking about abstract objects; the claim is that the only view that's *not inconsistent* with the linguistic intentions of ordinary speakers is the platonistic interpretation. But let me start at the beginning.

Note that the only options for what numerals like "3" might refer to (or purport to refer to) are physical objects, mental objects, and abstract objects. If an object *O* is a real thing, then it is either an ordinary physical object existing in the physical world; or a mental object, e.g., an idea in one of our heads (of course, if you're a materialist about the mind, then you'll want to say that mental objects are just a special kind of physical object, but let's not worry about whether this is true); or an abstract object. There just don't seem to be any other options.

Thus, if we can give empirical reasons for thinking that mathematical terms like "3" cannot be interpreted as referring (or purporting to refer) to physical or mental objects, then we will have good reason to adopt the semantic platonist view that we ought to interpret these terms as referring (or purporting to refer) to abstract objects. I'll say more about this later, but for now, let's proceed with the empirical reasons for rejecting physicalistic and psychologistic semantic theories. Let me begin by putting these two theories on the table for discussion:

> *Semantic Physicalism*: Ordinary mathematical sentences such as "2 + 2 = 4" and "3 is prime" are best interpreted as straightforward claims about ordinary physical objects. (John Stuart Mill [1843] endorsed a view of this general kind, but his view also had nonliteralist threads running through it.)
>
> *Semantic Psychologism*: Ordinary mathematical sentences such as "2 + 2 = 4" and "3 is prime" are best interpreted as straightforward claims about ordinary mental objects—that is, things like ideas that actually exist inside of our heads. (Brouwer [1912, 1948] and Heyting [1956] endorsed views that are in this general ballpark, but their views are interestingly distinct from semantic psychologism as I've defined it.)

I think these two theories are simply unacceptable. Let me start with semantic physicalism.

One problem with semantic physicalism is that if it were right, then it would be reasonable to worry that there just aren't enough objects in the world to make our mathematical theories true. Imagine a mathematics professor teaching Euclid's proof that there are infinitely many prime numbers, and imagine a student raising her hand with the following objection: "There couldn't be infinitely many prime numbers, because my physics professor told me that there are only finitely many physical objects in the whole universe."

Or to make the problem even more vivid, imagine that after being taught Cantor's theorem a student said, "There couldn't be infinitely many transfinite cardinals, because my physics professor assures me that there just aren't that many physical objects in the universe."

It seems reasonable to think that these two students just *don't understand*; they don't understand what the two proofs are supposed to show. For Euclid's and Cantor's proofs, it doesn't matter how many physical objects there are. The only reasonable conclusion we can draw from this, I think, is that the two theorems should not be interpreted as being about physical objects.

Here's a second argument against semantic physicalism: when we apply this semantic theory to set theory, we get the result that expressions that are supposed to refer to sets are supposed to refer to piles of physical stuff. But this can't be right, because corresponding to every pile of physical stuff—indeed, every individual physical object—there are infinitely many sets. Corresponding to a ball, for instance, is the set containing the ball, the set containing its molecules, the set containing that set, and so on. Clearly, these sets are not supposed to be purely physical objects, because (a) they are all supposed to be distinct from one another, and (b) they all share the same physical base (i.e., they're all made of the same matter and have the same spatiotemporal location). Thus, there must be something nonphysical about these sets, over and above the physical base that they all share. Or more precisely, the linguistic terms of set theory that are supposed to refer to these sets are not supposed to refer to piles of physical stuff.

These arguments show that there is no plausible way to interpret ordinary mathematical claims as being claims about physical objects. For facts about how many physical objects there are in the universe are completely irrelevant to ordinary claims about how many mathematical objects there are.

Let's move on now to semantic psychologism—to the view that ordinary mathematical sentences are supposed to be claims about mental objects like ideas in our heads.

If this were right, then it would be reasonable to worry that there aren't enough mental objects in the world to make our mathematical theories true. But this *isn't* reasonable; for instance, for Euclid's and Cantor's proofs, it doesn't *matter* how many mental objects there are in the universe. So the two theorems should not be interpreted as being about actual mental objects that exist in our heads.

There are other problems with semantic psychologism (see, e.g., Frege 1884), but instead of running through other arguments, let's reconsider the argument already given, to make sure that its power isn't overlooked. First, the worry here is *not* that humans can't conceive of an infinite set. The worry has to do with the number of mental objects (e.g., distinct number-ideas) that are *actually residing* in human heads. Semantic psychologism implies that in order for standard arithmetical theories such as Peano Arithmetic (PA) to be true, there must be an infinite number of these mental objects. But this just isn't true. If you're worried that PA might be false because there aren't enough actual ideas to go around, then that just shows that you don't understand what PA says. The conclusion we should draw here is that semantic psychologism is false.

Second, one might worry that the above argument is directed at a silly or trivial version of semantic psychologism that no one would ever endorse. But there is no way to get rid of the silliness without altering the view in a way that makes it no longer a version of semantic psychologism at all. Suppose, for instance, that someone said something like this:

> Psychologism isn't the view that mathematics is about *actual* ideas that exist inside of human heads. It's the view that mathematics is about what it's *possible* to do in our heads. For instance, to say that there are infinitely many prime numbers is not to say that there really exists an infinity of prime-number ideas inside of human heads; it's to say that it's possible to construct infinitely many prime numbers in our heads.

But this isn't a version of semantic psychologism at all, so it's no defense against the above objection. Semantic psychologism is the view that mathematical claims are about mental objects. The above view rejects this, and so it's not a version of semantic psychologism. Rather, it's a version of *nonliteralism*; in other words, it rejects the above thesis that when we say things like "3 is prime," we're speaking literally; on the view in question, "3 is prime" doesn't really say that a certain *object* (namely, 3) is prime; rather, it says something about what it's possible for humans to do. But as an empirical hypothesis about what people actually mean when they utter sentences like "3 is prime," this is just really implausible; there's no evidence that people really mean to say things like this when they utter sentences like "2 + 2 = 4" and "3 is prime."

If we remain clear on what semantic psychologism actually says, then the view is crazy, and the above argument shows that. And it's important to remember that the claim here is entirely semantic. None of this is to deny the ontological thesis that there *are* number-ideas in our heads. I take it that this is entirely obvious. What the above argument shows is that numerals like "3" shouldn't be taken to refer to these ideas, and sentences like "3 is prime" shouldn't be taken to be claims about these ideas.

Similarly, it should also be clear that studies that aim to show that our mathematical ideas originate in our brains (I'm thinking here of the work of people like Stanislas Dehaene) are completely irrelevant to a defense of semantic psychologism. It may be true that our mathematical ideas originate in our brains, and that platonic heaven didn't need to exist in order for us to come up with all of the mathematics that we have come up with; but it just doesn't follow that numerals like "3" are supposed to refer to things inside our heads. An analogy here is the God case; you might think that our God thoughts originate in our brains, and that God didn't need to exist in order for us to come up with these thoughts; but it doesn't follow that the term "God" is supposed to refer to something inside our heads, and in fact, it is entirely obvious that it's *not* supposed to refer to something inside our heads; it's supposed to refer to a creator of the world (you should admit that this is true whether you believe in the existence of such a creator or not).

Finally, it's worth noting that the above argument against semantic psychologism should not be taken as an argument against *intuitionism*. It is often thought that intuitionism is a form of psychologism, but this is a mistake. True, many intuitionists—most notably, Brouwer (1912, 1948) and Heyting (1956)—have also endorsed psychologistic views. But intuitionism is perfectly consistent with platonism and other antipsychologistic views, and psychologism is consistent with a rejection of intuitionism.

In any event, we now have arguments against semantic physicalism and semantic psychologism, and if we combine these arguments with the above argument for premise (1), we get an argument for semantic platonism, i.e., for the claim that sentences like "3 is prime" are best interpreted as being about abstract objects (or at least purporting to be about abstract objects). The argument goes like this:

(1) Ordinary mathematical sentences like "2 + 2 = 4" and "3 is prime" should be interpreted at face value. Thus, "3 is prime" says that a certain object (namely, the number 3) has a certain property (namely, the property of being prime).
(2) Given that ordinary mathematical sentences should be interpreted as making claims about certain objects (namely, numbers), we can't interpret them as being about physical or mental objects, so we have to interpret them as being about abstract objects (or more precisely, as *purporting* to be about abstract objects). Therefore,
(3) Semantic platonism is true. In other words, ordinary mathematical sentences like "2 + 2 = 4" and "3 is prime" are (or purport to be) claims about abstract objects.

Now, you might object here that just as there are reasons to resist semantic physicalism and semantic psychologism, so too there are reasons to resist semantic platonism. For you might think it's implausible that ordinary people intend to be speaking of abstract objects when they say things like "3 is prime." But semantic platonists don't need to say that people have such intentions, and indeed, they *shouldn't* say this. What they should say is that (a) people are best interpreted as speaking literally when they say things like "3 is prime," and so these sentences have to be taken as being about objects (in particular, numbers); and (b) our semantic intentions are incompatible with semantic physicalism and semantic psychologism, and so there is no way to interpret us as talking about physical or mental objects when we say things like "3 is prime" (this is what the above arguments show); and (c) there's nothing in our intentions that's incompatible with semantic platonism; and so (d) even if people don't have a positive intention to refer to abstract objects when they say things like "3 is prime," the best interpretation of these utterances has it that they *are* about abstract objects (or at least that they *purport* to be about such objects).

We're done: we have a purely empirical argument for semantic platonism.

From Levelheaded Empirical Semantics to Crazy Ontology

Now let's use this argument to argue for the *ontological* thesis that platonism is true. The argument goes like this:

(i) Semantic platonism is true—i.e., ordinary mathematical sentences like "2 + 2 = 4" and "3 is prime" are (or purport to be) claims about abstract objects. Therefore,
(ii) Mathematical sentences like "2 + 2 = 4" and "3 is prime" could be true only if platonism were true—i.e., only if abstract objects existed. But
(iii) Mathematical sentences like "2 + 2 = 4" and "3 is prime" *are* true. Therefore,
(iv) Platonism is true.

I already argued for (i). But (ii) seems to follow immediately from (i). Think first of the sentence "Mars is red"; this couldn't be true unless Mars existed. And likewise, given (i), "3 is prime" couldn't be true unless an abstract object existed, namely, the number 3. Finally, (iii) seems obvious, and when we combine (iii) with (ii), it implies platonism. So our levelheaded empirical semantic investigation seems to have led us to a crazy ontological thesis. We have two seemingly obvious premises—namely, semantic platonism and the truth of mathematics—and they lead to the crazy conclusion that there's a platonic realm of nonphysical, nonmental, nonspatiotemporal objects. How did that happen?

Well, one analysis of how it happened is that premise (iii) is a lot more controversial than it seems. For given our platonistic semantics, the claim that mathematical sentences like "2 + 2 = 4" are literally true is tantamount to the claim that platonism is true. We can bring this point out by noting that one might also argue as follows:

(i) Semantic platonism is true—i.e., ordinary mathematical sentences like "2 + 2 = 4" and "3 is prime" are (or purport to be) claims about abstract objects. Therefore,

(ii) Mathematical sentences like "2 + 2 = 4" and "3 is prime" could be true only if platonism were true—i.e., only if abstract objects existed. But

(not-iv) Platonism *isn't* true: there's no platonic heaven, and there are no such things as nonphysical, nonmental, nonspatiotemporal abstract objects. Therefore,

(not-iii) Mathematical sentences like "2 + 2 = 4" and "3 is prime" are *not true*.

You might think this argument is just as compelling as the last argument. It too has extremely plausible premises and a crazy conclusion. But it's not clear which argument is better.

We can call the view expressed in the conclusion of this argument *fictionalism*. But fictionalists do *not* think that mathematics is perfectly analogous to novel writing. That's not the view. The view is simply that mathematical sentences aren't literally true because (a) they're supposed to be about abstract objects and (b) there are no such things as abstract objects. So a better name would be *not-literally-true-ism*. In any event, this view was first introduced by Field (1980), and it has been further developed by Rosen (2001), Yablo (2002), Leng (2010), and myself (1998).)

Which of these two arguments should we endorse? Well, there's also a *third* argument here that's a bit safer than either of the first two and is, I think, very interesting. I would actually endorse it. It goes like this:

(i) Semantic platonism is true—i.e., ordinary mathematical sentences like "2 + 2 = 4" and "3 is prime" are (or purport to be) claims about abstract objects. Therefore,

(ii) Mathematical sentences like "2 + 2 = 4" and "3 is prime" could be true only if platonism were true—i.e., only if abstract objects existed. Therefore,

(iii*) Either platonism or fictionalism is true.

Insofar as platonism and fictionalism are both crazy, we seem to have a purely empirical argument here for the claim that *something* crazy is going on in the philosophy of mathematics. If our empirical semantic theory is right, then our only options are platonism and fictionalism. And as far as I can see, there's no good reason for favoring either of these views over the other.

I suspect that for a lot of mathematicians, the idea that "2 + 2 = 4" is untrue is pretty hard to swallow. If that's how you feel, then you can endorse platonism—though, for the life of me, I don't know how you could justify that. But perhaps it will give you some solace to learn that according to fictionalism—or at any rate, the best versions of fictionalism—it isn't just mathematics that turns out to be untrue. According to the version of fictionalism that I favor, empirical theories such as Quantum Mechanics are untrue as well, because these theories refer to abstract mathematical objects. So mathematicians are no worse off in this regard than anyone else is. Now, maybe it bothers you to think that our mathematical and scientific theories are untrue. But it doesn't bother me. The trick is to notice that (a) according to fictionalism, our mathematical and scientific theories are *virtually* true, or *for-all-practical-purposes* true, or some such thing (because they're such that they *would* be true if there were abstract objects), and (b) if fictionalism is true, then it's this virtual truth, or for-all-practical-purposes truth, that's really important. Literal truth, on this view, just isn't very important; it isn't to be valued; and so it just doesn't matter if our mathematical and scientific theories aren't literally true.

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