PROBLEMS AND SOLUTIONS

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Proposed problems should be submitted online at
americanmathematicalmonthly.submittable.com/submit.
Proposed solutions to the problems below should be submitted by August 31, 2019 via the same link. More detailed instructions are available online. Proposed problems must not be under consideration concurrently at any other journal nor be posted to the internet before the deadline date for solutions. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.

PROBLEMS

12104. *Proposed by Joe Buhler, Larry Carter, and Richard Stong, Center for Communications Research, San Diego, CA.* Consider a standard clock, where the hour, minute, and second hands all have integer lengths and all point straight up at noon and midnight. Is it possible for the ends of the hands to form, at some time, the vertices of an equilateral triangle?

12105. *Proposed by Gary Brookfield, California State University, Los Angeles, CA.* Let r be a real number, and let $f(x) = x^3 + 2rx^2 + (r^2 - 1)x - 2r$. Suppose that f has real roots a, b, and c. Prove $a, b, c \in [-1, 1]$ and $|\arcsin a| + |\arcsin b| + |\arcsin c| = \pi$.

12106. *Proposed by Hideyuki Ohtsuka, Saitama, Japan.* For any positive integer n, prove

$$\sum_{k=1}^{n} \binom{n}{k} \sum_{1 \le i \le j \le k} \frac{1}{ij} = \sum_{1 \le i \le j \le n} \frac{2^n - 2^{n-i}}{ij}.$$

12107. *Proposed by Cornel Ioan Vălean, Teremia Mare, Romania.* Prove

$$\int_0^1 \int_0^1 \frac{1}{\sqrt{1+x^2}\sqrt{1+y^2}(1-x^2y^2)}\, dx\, dy = G,$$

where G is Catalan's constant $\sum_{n=1}^{\infty} (-1)^{n-1}/(2n-1)^2$.

12108. *Proposed by Yifei Pan and William D. Weakley, Purdue University Fort Wayne, Fort Wayne, IN.* Let n be a positive integer, and let $\beta_1, \dots, \beta_n$ be indeterminates over a field F. Let M be the n-by-n matrix whose i, j-entry m_{ij} is given by $m_{ij} = \beta_i$ when $i = j$ and $m_{ij} = 1$ when $i \ne j$. Show that the polynomial $\det(M)$ is irreducible over F.

12109. *Proposed by George Stoica, Saint John, NB, Canada.* Let f be a function on $[0, \infty)$ that is nonnegative, bounded, and continuous. For $a > 0$ and $x \ge 0$, let $g(x) = \exp\left(\int_0^a \log(1 + xf(s))\, ds\right)$. For $0 < p < 1$, prove

$$\int_0^a f^p(x)\, ds = \frac{p \sin(p\pi)}{\pi} \int_0^{\infty} \frac{\log g(x)}{x^{p+1}}\, dx.$$

doi.org/10.1080/00029890.2019.1574184