Solutions to FE Exam "Dynamics" Review Problems

Prepared by
Stephen F. Felszeghy
CSULA Emeritus Professor of Mechanical Engineering

Presented here are my solutions to the "Dynamics" review problems that used to be available online, until the end of 2017, on the website for the book: Beer and Johnston, *Vector Mechanics for Engineers, Statics and Dynamics*, Ninth Edition, 2010, at: http://highered.mcgraw-hill.com/sites/0073529400/information_center_view0/ . Before these review problems went "Out of Print," I downloaded and collected them in the following "problems.pdf" file: http://www.calstatela.edu/sites/default/files/users/u28426/felszeghy/problems.pdf .

My solutions, which you will find below, are for the review problems that are associated with the following chapters and topics in the above book:

Chpt. 11: Kinematics of Particles
Chpt. 12: Kinetics of Particles: Newton's Second Law
Chpt. 13: Kinetics of Particles: Energy and Momentum Methods
Chpt. 14: Systems of Particles
Chpt. 15: Kinematics of Rigid Bodies
Chpt. 16: Plane Motion of Rigid Bodies: Forces and Accelerations
Chpt. 17: Plane Motion of Rigid Bodies: Energy and Momentum Methods
Chpt. 19: Mechanical Vibrations

As I mentioned in class, some of the formerly available online problem statements had errors in them, and some online solutions and answers were wrong! For this reason, I have included in the above mentioned "problems.pdf" file a list of errors and corrections. Although I shared the errors and corrections with McGraw-Hill, the company never made any corrections to its Website.

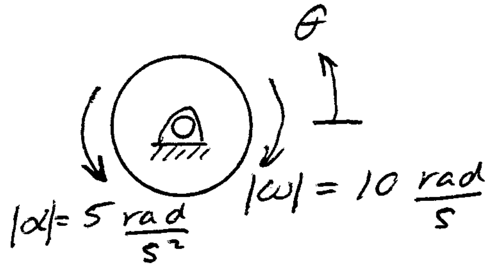
The formerly available online problems were not numbered; they were identified by chapter numbers only. For this reason, when I downloaded the online problems, I numbered them consecutively in a decimal format, XX.X, where XX refers to the chapter number, and X stands for the sequence number. All the downloaded problems numbered this way are included in the above mentioned "problems.pdf" file under the heading: "Part 1, FE Exam Review, Online Problems and Solutions."

My own solutions, which you will find below, follow the problem numbering scheme I established above. I included sketches in my solutions to allow you to identify more easily the problems to which my solutions apply.

I wish you all the best on your computer-based FE exam!

Chpt. 11

11.1



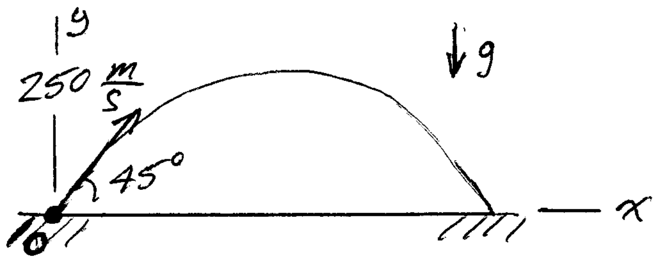
$\omega_0 = -10\ \frac{rad}{s}$

$\alpha = 5\ \frac{rad}{s^2}$

$\omega = \omega_0 + \alpha t = -10 + 5t$

$\omega = 0$ at $t = 2\,s$ ← Ans.

11.2



$v_y = v_0 \sin\theta - gt$

$v_y = 0$ at $t_1 = \frac{v_0 \sin\theta}{g}$

$= \frac{250 \sin 45°}{9.81}$

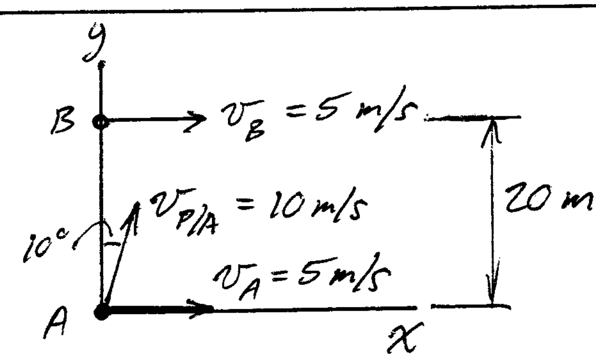
$= 18.02\ s$

$y = 0$ at $t_2 = 2t_1 = 36.04\,s$

x @ $t_2 = 36.04\,s$:

$x = v_0 (\cos\theta) t_2 = 250(\cos 45°)36.04 = 6371\ m$ ← Ans.

11.3



A and B remain fixed in moving x-y axes attached to A.

Puck motion in y-direction:

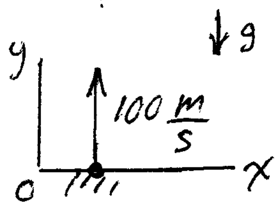
$y = v_{P/A}(\cos 10°)t$

Puck reaches line of $\underline{v}_B$ at $t = \frac{20}{10 \cos 10°} = 2.031\ s$

x @ $t = 2.031\,s$: $x = v_{P/A}(\sin 10°)t = 10(\sin 10°)2.031$

$= 3.53\ m$ ← Ans.

11.4

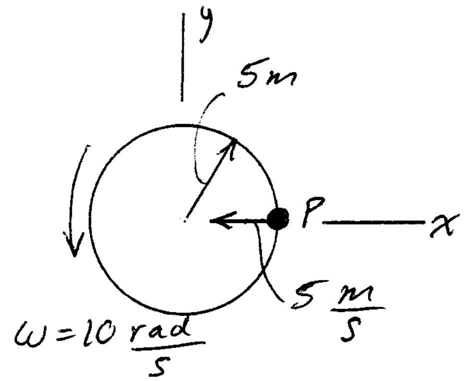


$v = v_0 + a_0 t \qquad a_0 = -g$

$v = 0$ at $t = \frac{v_0}{g} = \frac{100}{9.81} = 10.19\,s$ ← Ans.

Note: $y = y_{max}$ when $v = 0$: $y_{max} = v_0 t - a_0 \frac{t^2}{2} = 509.7\ m$

11.5



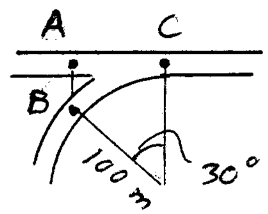
Use polar coordinates:

$$a_r = \ddot{r} - r\dot{\theta}^2$$
$$= 0 - (5)(10)^2 = -500 \ \frac{m}{s^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$
$$= 0 + (2)(-5)(10) = -100 \ \frac{m}{s^2}$$

$$\underline{a} = -500\,\underline{e}_r - 100\,\underline{e}_\theta = -500\,\underline{i} - 100\,\underline{j}, \ \frac{m}{s^2} \leftarrow \text{Ans.}$$

11.6



Time for B to arrive at C:

$$t = \frac{s_B}{v_B} = \frac{(\pi/6)100}{\left(\frac{90\times 1000}{3600}\right)} = 2.094 \ s$$

Time for A to arrive at C:

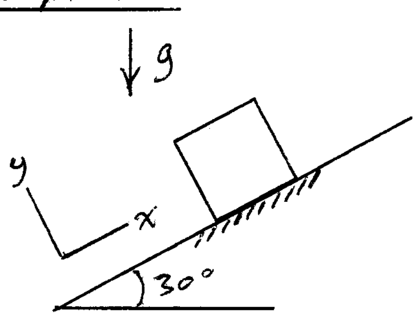
$$t = \frac{s_A}{v_A} = \frac{100 \sin 30^\circ}{\left(\frac{100\times 1000}{3600}\right)} = 1.800 \ s$$

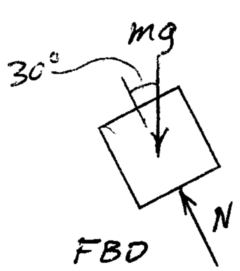
So A gets to C first and B gets to C by distance

B is behind A when

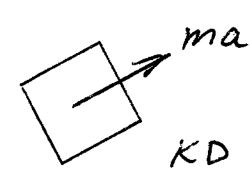
$$d = (2.094 - 1.8)\left(\frac{100\times 1000}{3600}\right)$$
$$= 8.17 \ m \leftarrow \text{Ans}$$

12.1 Chpt. 12





=



$$\Sigma F_x = ma \Rightarrow -mg \sin 30^\circ = ma$$

$$a = -9.81 \sin 30^\circ = -4.905 \ \frac{m}{s^2}$$

$$v = v_0 + at = 5 - 4.905\,t$$

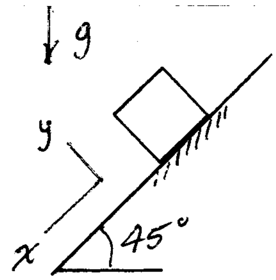
$$v = 0 \text{ when } t = \frac{5}{4.905} = 1.019 \ s$$

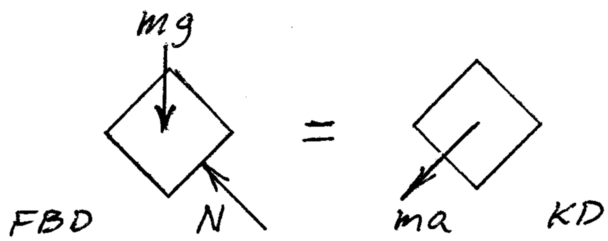
$$x \ @ \ t = 1.019 s: \quad x = v_0 t + \frac{at^2}{2} = (5)(1.019) - 4.905\frac{(1.019)^2}{2} = 2.55 \ m$$

Ans.

INSERT AFTER 2

12.2





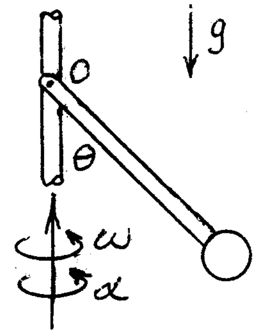
$$\Sigma F_x = ma$$

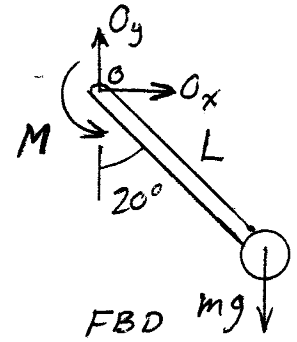
$$\cancel{m}g \sin 45^\circ = \cancel{m}a$$

$$a = (9.81)\cos 45^\circ = 6.94\ \frac{m}{s^2}$$ ← Ans.

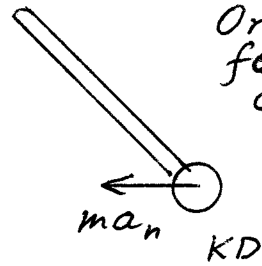
③

12.3





=



Note: Only in-plane forces and couple are shown.

$$\circlearrowleft \Sigma M_o = -ma_n L\cos 20^\circ$$

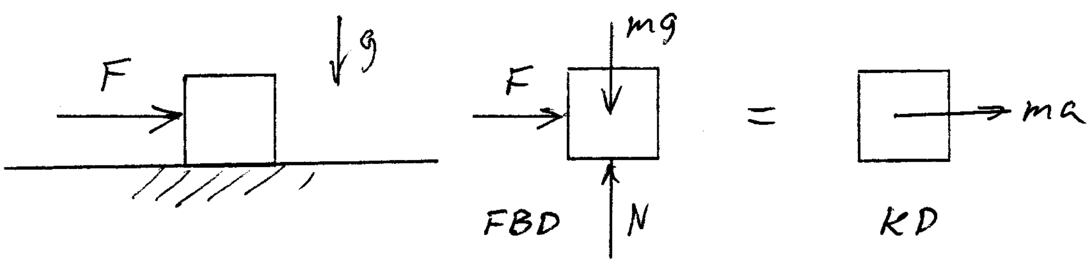
$$M - mgL\sin 20^\circ = -m(\omega^2 L\sin 20^\circ)L\cos 20^\circ$$

$$M = (5)(9.81)(2\sin 20^\circ) - (5)\left(\frac{50\times 2\pi}{60}\right)^2(2\sin 20^\circ)(2\cos 20^\circ)$$

$$M = -142.7\ \text{N}\cdot\text{m}$$

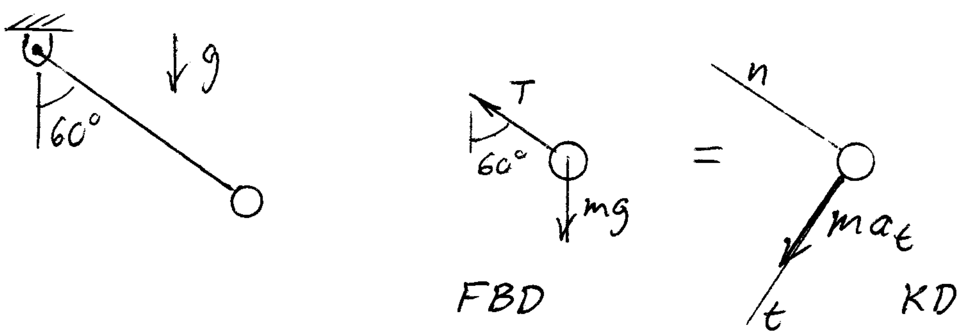
$$\underline{M} = 142.7\ \text{N}\cdot\text{m} \circlearrowright \quad \longleftarrow \text{Ans.}$$

12.4



$$F = ma = (4)(15) = 60\ \text{N} \quad \longleftarrow \text{Ans.}$$

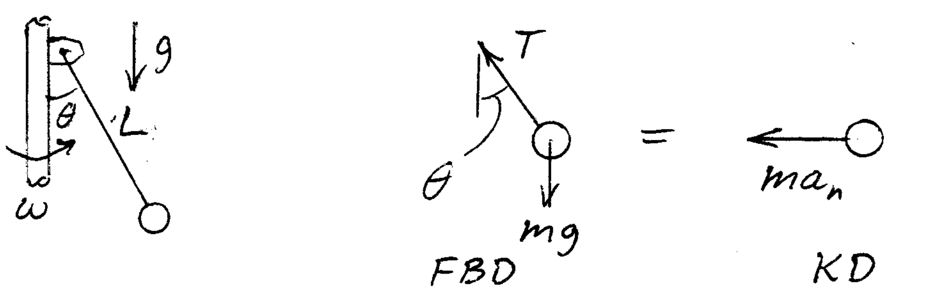
12.5

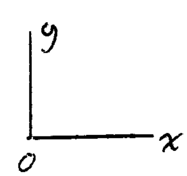


$$\Sigma F_n = 0 \implies T - mg\cos 60^\circ = 0$$

$$T = mg\cos 60^\circ = (3)(9.81)\cos 60^\circ = 14.72\ \text{N} \quad \longleftarrow \text{Ans.}$$

12.6





(Cont'd next page.)

$$\Sigma F_x = -ma_n \Rightarrow -T\sin\theta = -ma_n \quad (1)$$

$$\Sigma F_y = 0 \Rightarrow T\cos\theta - mg = 0 \quad (2)$$

Eliminate T between (1) & (2):

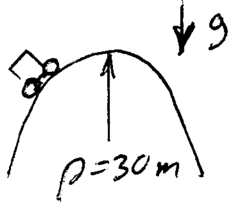
$$\tan\theta = \frac{\cancel{m}a_n}{\cancel{m}g}$$

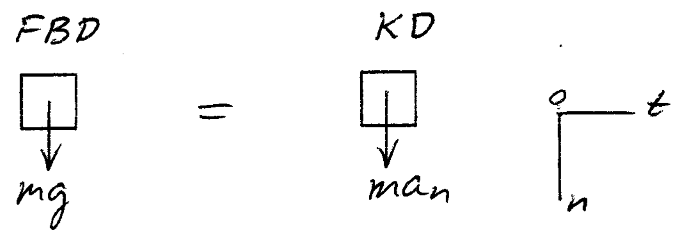
$$= \frac{\omega^2 L\sin\theta}{g}$$

$$\cos\theta = \frac{g}{\omega^2 L} = \frac{9.81}{\left(\frac{20\times 2\pi}{60}\right)^2 4} = 0.559$$

$$\theta = 56.0^\circ \longleftarrow \text{Ans.}$$

12.7





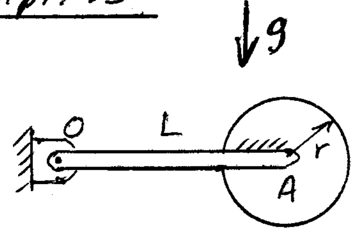
$$\Sigma F_n = ma_n$$

$$\cancel{m}g = \cancel{m}\frac{v^2}{\rho}$$

$$v^2 = \rho g = (30)(9.81)$$

$$v = 17.1\ m/s \longleftarrow \text{Ans.}$$

13.1 Chpt. 13



$$0 = \Delta T + \Delta V_g$$

$$\Delta T = \frac{1}{2} I_o \omega_2^2$$

$$= \frac{1}{2}\left[\frac{1}{3}m_r L^2 + \frac{1}{2}m_d r^2 + m_d L^2\right]\omega_2^2$$

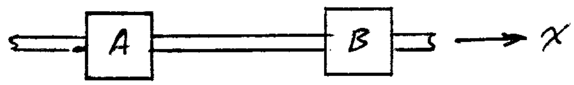
$$= \frac{1}{2}\left[\frac{1}{3}\frac{10}{9.81}1^2 + \frac{1}{2}10(0.3)^2 + 10(1)^2\right]\omega_2^2$$

$$= 5.39\,\omega_2^2$$

$$\Delta V_g = -m_r g\frac{L}{2} - m_d g L = -10(0.5) - 10(9.81)(1) = -103.1$$

$$\Delta T = -\Delta V_g \Rightarrow \omega_2^2 = 19.11, \quad \omega_2 = 4.37, \quad v_A = \omega_2 1 = 4.37\,\frac{m}{s} \longleftarrow \text{Ans.}$$

13.2



Conservation of total linear momentum:

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$(10)(10) + (20)(-15) = 10 v_A' + 20 v_B'$$

$$-200 = 10 v_A' + 20 v_B' \quad (1)$$

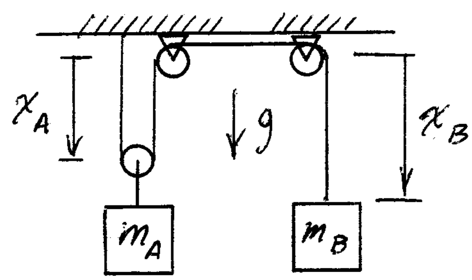
$$e = \frac{v_B' - v_A'}{v_A - v_B} \Rightarrow 0.6 = \frac{v_B' - v_A'}{10-(-15)} \Rightarrow 15 = -v_A' + v_B' \quad (2)$$

$$(1) - 20 \times (2) \Rightarrow -200 - 300 = 10 v_A' + 20 v_A'$$

$$30 v_A' = -500$$

$$v_A' = -16.67 \ \frac{m}{s} \quad \leftarrow \text{Ans.}$$

13.3

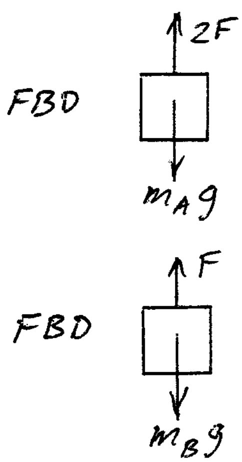


Cable length is const. Therefore,

$$2x_A + x_B + \text{const.} = \text{const.}$$

$$2\Delta x_A = -\Delta x_B \quad (1)$$

$$2\dot{x}_A = -\dot{x}_B \quad (2)$$



Work-energy:

$$m_A g \Delta x_A - 2F\Delta x_A = \frac{1}{2} m_A \dot{x}_A^2 \quad (3)$$

$$m_B g \Delta x_B - F\Delta x_B = \frac{1}{2} m_B \dot{x}_B^2 \quad (4)$$

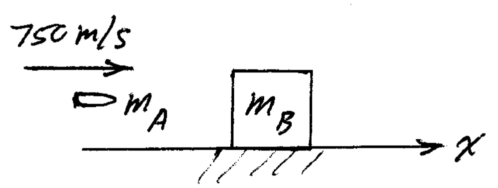
Substitute (1) into (3), and (2) into (4), and add resulting equations

$$(-m_A g/2 + m_B g)\Delta x_B = \frac{1}{2}(m_A \dot{x}_A^2 + m_B 4\dot{x}_A^2)$$

$$9.81(-15+40)(10) = (15+80)\dot{x}_A^2$$

$$\dot{x}_A^2 = 25.82, \quad \dot{x}_A = 5.08 \ \frac{m}{s} \quad \leftarrow \text{Ans.}$$

13.4



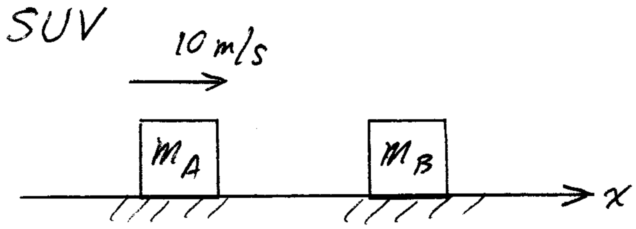
Conservation of total linear momentum:

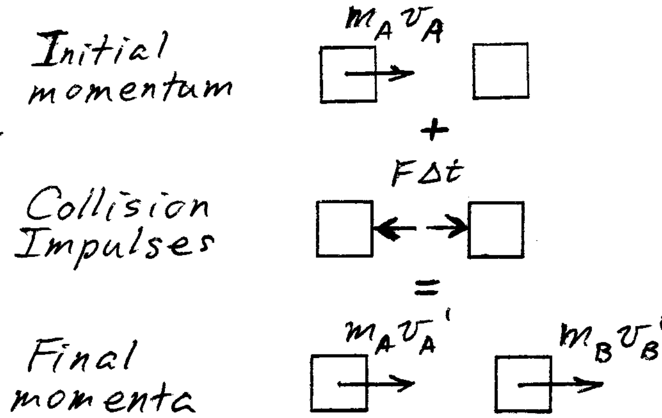
$$m_A v_A + \cancel{m_B v_B}^{\,0} = (m_A + m_B) v'$$

$$v' = \frac{m_A v_A}{m_A + m_B} = \frac{(0.015)(750)}{10.015}$$

$$v' = 1.123 \ \frac{m}{s} \quad \leftarrow \text{Ans}$$

13.5





$$e = \frac{v_B' - v_A'}{v_A - \cancel{v_B}^{\,0}}$$

$$0.7 = \frac{v_B' - v_A'}{10} \quad (1)$$

Conservation of total linear momentum:

$$m_A v_A + m_B \cancel{v_B}^{\,0} = m_A v_A' + m_B v_B'$$

$$(6000)(10) = 6000 v_A' + 4000 v_B'$$

or $$10 = v_A' + \frac{2}{3} v_B' \quad (2)$$

Add (1) & (2): $$17 = \frac{5}{3} v_B'$$

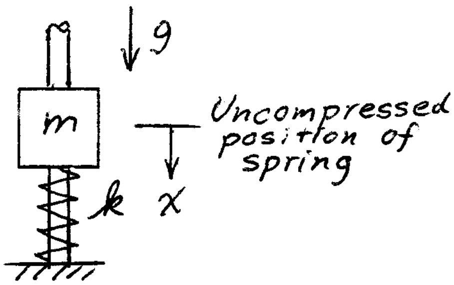
$$v_B' = 10.2 \ \text{m/s}$$

Apply impulse-momentum eq. for B:

$$F\Delta t = m_B v_B'$$

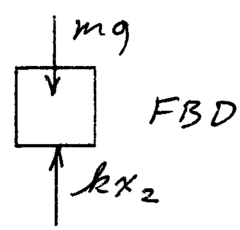
$$F = \frac{m_B v_B'}{\Delta t} = \frac{(4000)(10.2)}{0.3} = 136 \ \text{kN} \longleftarrow \text{Ans.}$$

13.6



$k = 0.1 \ \text{kN/mm}$ (NEW)

$x_1 = 0.02 \ \text{m}$, $v_1 = 0 \ \text{m/s}$.
$v_2 = v_{max}$ when $a_2 = 0$.



$$kx_2 = mg$$

$$x_2 = \frac{(10)(9.81)}{10^5} = 98.1 \times 10^{-5} \ \text{m}$$

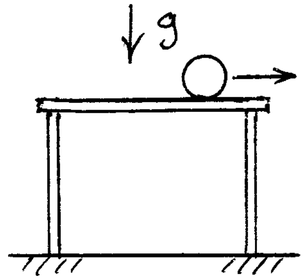
$$0 = \Delta T + \Delta V_g + \Delta V_e$$

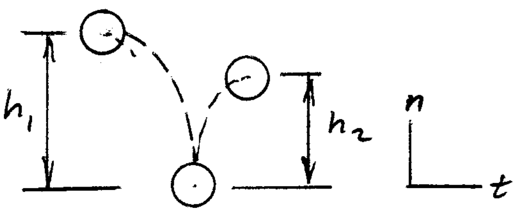
$$0 = \frac{1}{2} m v_{max}^2 - mg(x_2 - x_1) + \frac{1}{2} k (x_2^2 - x_1^2)$$

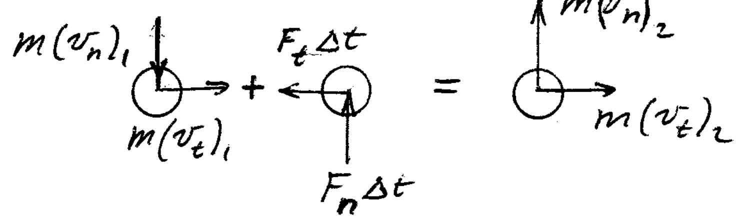
$$v_{max}^2 = \frac{2}{10}\left[-(10)(9.81)(0.02 - 98.1\times10^{-5}) + \frac{1}{2}10^5(0.02^2 - 98.1^2\times10^{-10})\right]$$

$$v_{max}^2 = 3.617 \Longrightarrow v_{max} = 1.902 \ \frac{\text{m}}{\text{s}} \longleftarrow \text{Ans.}$$

13.7







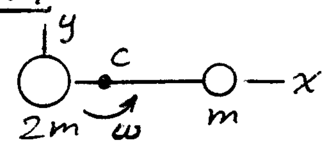
$$0 = \Delta T + \Delta V_g \Longrightarrow \quad \tfrac{1}{2} m[(v_n)_1]^2 = mgh_1 \quad (1)$$

$$\tfrac{1}{2} m[(v_n)_2]^2 = mgh_2 \quad (2)$$

Solve (1) and (2) for $(v_n)_1$ and $(v_n)_2$, and substitute in eq. below:

$$e = \frac{(v_n)_2}{(v_n)_1} = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{0.8}{1}} = 0.894 \longleftarrow \text{Ans.}$$

14.1 Chpt. 14



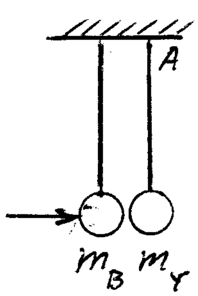
Center of mass: $x_c = \frac{3m}{3m}$

$= 1\ m$

$$H_c = 2 \times m(2\omega) + 1 \times 2m(1\omega)$$

$$= 6m\omega = 6(1)(5) = 30\ \frac{kg \cdot m^2}{s} \longleftarrow \text{Ans.}$$

14.2 See middle p. 8



Conserv. of total linear momentum:

$$m_B v_B + \cancel{m_Y v_Y}^{\,0} = m_B v_B' + m_Y v_Y'$$

$$(0.5)(6) = 0.5 v_B' + (1) v_Y' \quad (1)$$

$$e = \frac{v_Y' - v_B'}{v_B}$$

$$1 = \frac{v_Y' - v_B'}{6} \Longrightarrow 6 = -v_B' + v_Y' \quad (2)$$

$$(1) + 0.5 \times (2) \Longrightarrow \quad 6 = 1.5 v_Y' \Longrightarrow v_Y' = 4\ \frac{m}{s}$$

$$H_A = 3 \times m_Y v_Y' = 3 \times (1)(4) = 12\ kg \cdot \frac{m^2}{s} \longleftarrow \text{Ans.}$$

14.3

$m = 5\ kg \qquad \underline{r} = 10\underline{i} - 2\underline{j} + 5\underline{k},\ m$

$\underline{v} = 3\underline{i} + 2\underline{j} - 5\underline{k},\ m/s$

$$\underline{H}_o = \underline{r} \times m\underline{v} = 5 \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 10 & -2 & 5 \\ 3 & 2 & -5 \end{vmatrix}$$

$$= 5\left[\underline{i}(10-10) - \underline{j}(-50-15) + \underline{k}(20+6)\right]$$

$$= 325\underline{j} + 130\underline{k},\ \frac{kg\cdot m^2}{s} \longleftarrow \text{Ans.}$$

14.4 See (7) bottom $\Longrightarrow v_Y' = 4\ \frac{m}{s}$

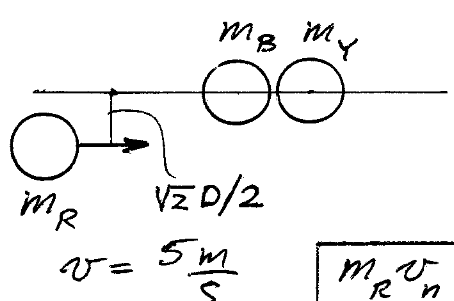
$m_Y: \quad 0 = \Delta T + \Delta V_g$

$$0 = \left[0 - \tfrac{1}{2} m_Y (v_Y')^2\right] + m_Y g\left[y_2 - 0\right]$$

$$y_2 = \frac{\frac{1}{2}\cancel{m_Y}(4)^2}{\cancel{m_Y}\, g} = 0.815\ m \longleftarrow \text{Ans.}$$

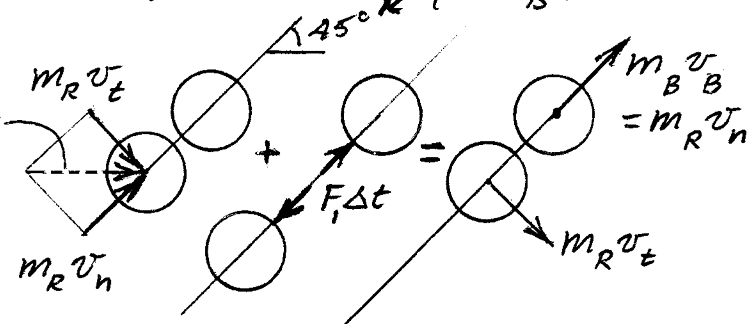
y, x, o

14.5

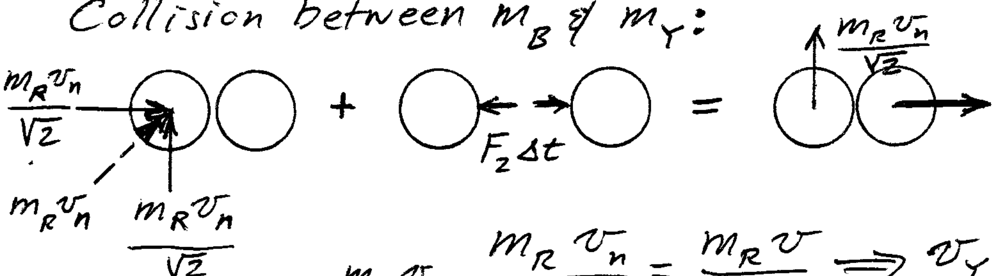


Collision between m_R & m_B:

$$\boxed{m_R v_n = \frac{m_R v}{\sqrt{2}}}$$



Collision between m_B & m_Y:

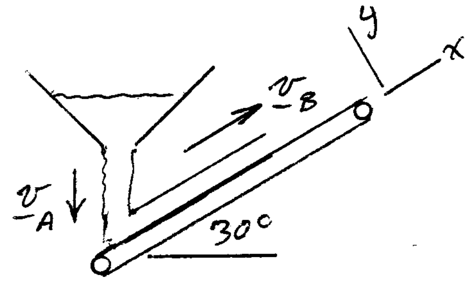


$$m_Y v_Y = \frac{m_R v_n}{\sqrt{2}}$$

$$m_Y v_Y = \frac{m_R v_n}{\sqrt{2}} = \frac{m_R v}{2} \Longrightarrow v_Y = \frac{v}{2} = 2.5\ \frac{m}{s} \longleftarrow \text{Ans.}$$

(9)

14.6

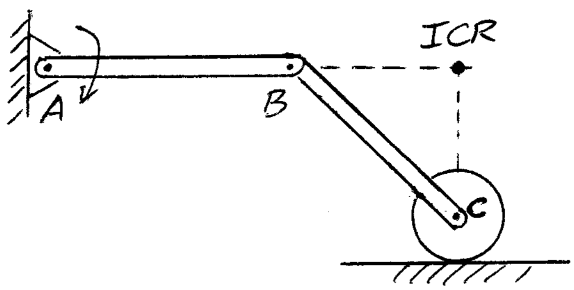


$$\Sigma \underline{F} = \frac{dm}{dt}\left(\underline{v}_B - \underline{v}_A\right)$$

$$\Sigma F_x = \frac{20000}{9.81 \times 3600}\left(0.5 - (-0.5 \sin 30°)\right)$$

$$= 0.425 \text{ N} \quad \leftarrow \text{Ans.}$$

15.1 Chpt. 15



$$\underline{\omega}_{AB} = 10 \frac{\text{rad}}{s} \curvearrowright$$

$$\underline{\alpha}_{AB} = 0 \frac{\text{rad}}{s^2}$$

$$\underline{a}_c = \underline{a}_B + \underline{a}_{c/B}$$

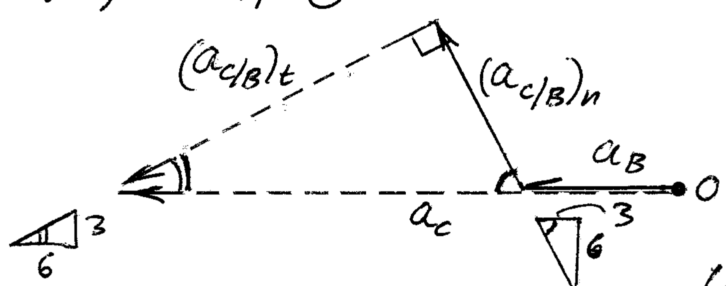
$$= \underline{a}_B + (\underline{a}_{c/B})_t + (\underline{a}_{c/B})_n \quad (1)$$

$$a_B = \omega_{AB}^2 \overline{AB} = (10)^2(0.6) = 60 \text{ m/s}^2$$

Using ICR of BC: $\omega_{Bc} = \dfrac{v_B}{0.3} = \dfrac{\omega_{AB}\overline{AB}}{0.3} = \dfrac{(10)(0.6)}{0.3} = 20 \circlearrowleft$

$$(a_{c/B})_n = \omega_{Bc}^2 \overline{BC} = (20)^2\sqrt{0.3^2+0.6^2} = 268.3 \frac{m}{s^2}$$

Vector diagram of (1):



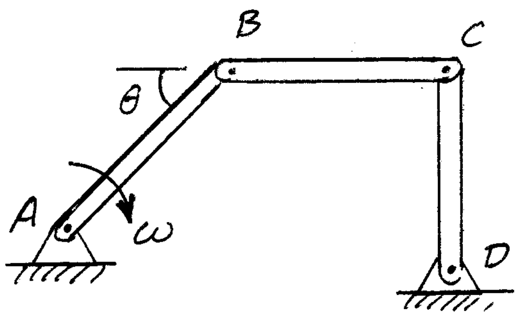
$$(a_{c/B})_t = (a_{c/B})_n \frac{6}{3}$$

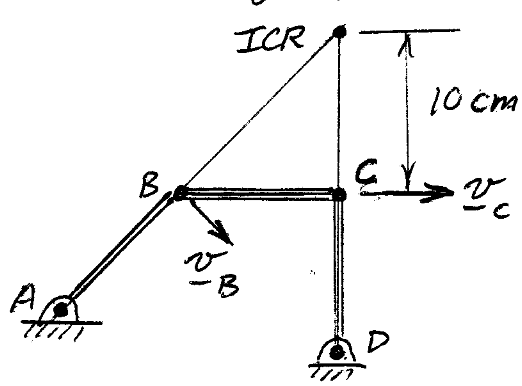
$$= 536.7 \frac{m}{s^2}$$

$$(a_{c/B})_t = \alpha_{BC} \overline{BC}$$

$$\alpha_{BC} = \frac{536.7}{\sqrt{0.45}} = 800 \frac{\text{rad}}{s^2} \curvearrowright \quad \leftarrow \text{Ans.}$$

15.2





$\leftarrow$ Ans.

15.3 See bottom of 9. $\underline{\omega}_{AB} = 5 \text{ rad/s} \curvearrowright$

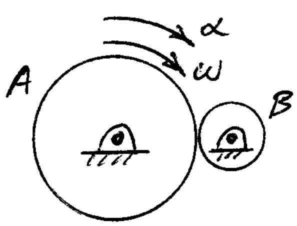
Using ICR of BC: $\omega_{BC} = \dfrac{v_B}{\sqrt{2}\,0.1} \curvearrowleft$

$$\omega_{BC} = \frac{\omega_{AB}\overline{AB}}{\sqrt{2}\,0.1}$$

$$v_c = \omega_{BC} 0.1 = \frac{\omega_{AB}\overline{AB}}{\sqrt{2}\,\cancel{0.1}}\cancel{0.1}$$

$$\omega_{CD} = \frac{v_c}{\overline{CD}} = \frac{\omega_{AB}}{\sqrt{2}}\frac{\overline{AB}}{\overline{CD}} = \frac{(5)(0.14)}{\sqrt{2}\,(0.20)} = 2.47 \text{ rad/s} \curvearrowright \leftarrow \text{Ans.}$$

15.4



Gear A: $d_A = 20\text{cm}$

Gear B: $d_B = 5\text{cm}$

$\omega_A = 20 \text{ rad/s}$

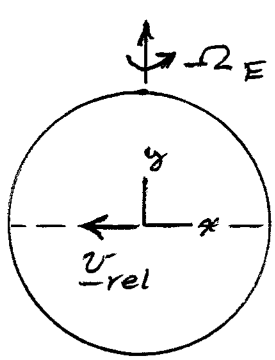
$\alpha_A = 4 \text{ rad/s}^2$

At point of engagement: $\alpha_A r_A = \alpha_B r_B$

$$\alpha_B = \alpha_A \frac{r_A}{r_B} = 4\left(\frac{10}{2.5}\right)$$

$$\alpha_B = 16 \frac{\text{rad}}{s^2} \curvearrowleft \leftarrow \text{Ans.}$$

15.5

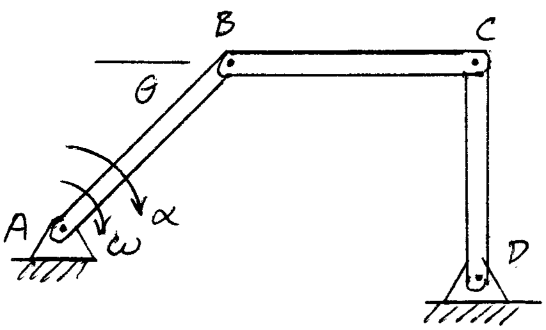


$$2\underline{\Omega}_E \times \underline{v}_{rel} = 2\Omega_E \underline{j} \times (-v_{rel}\,\underline{i})$$

$$= 2\Omega_E v_{rel} \underbrace{(\underline{j} \times -\underline{i})}_{\underline{k}}$$

$$= 2\Omega_E v_{rel}\,\underline{k} \leftarrow \text{Ans.}$$

15.6

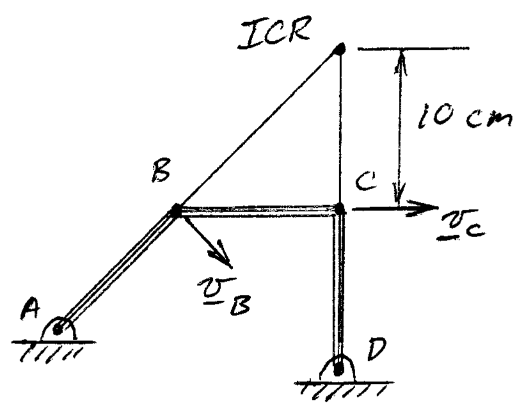


$\theta = 45^\circ$

$\underline{\omega}_{AB} = 5 \ \frac{rad}{s}$ ↻

$\underline{\alpha}_{AB} = 10 \ \frac{rad}{s^2}$ ↻

Using the ICR of BC:



$$\omega_{BC} = \frac{v_B}{\sqrt{2}\, 0.1}$$

$$\omega_{BC} = \frac{\omega_{AB}\,\overline{AB}}{\sqrt{2}\, 0.1} = \frac{(5)(.1414)}{\sqrt{2}\ 0.1}$$

$$= 5 \ \frac{rad}{s} \ \circlearrowleft$$

$$v_c = \omega_{BC}\, 0.1 = 0.5$$

$$v_c = \omega_{CD}\, \overline{CD}$$

$$\omega_{CD} = \frac{0.5}{0.2} = 2.5 \ \frac{rad}{s} \ \circlearrowright$$

$$\underline{a}_c = \underline{a}_B + \underline{a}_{c/B}$$

$$(\underline{a}_c)_t + (\underline{a}_c)_n = (\underline{a}_B)_t + (\underline{a}_B)_n + (\underline{a}_{c/B})_t + (\underline{a}_{c/B})_n \qquad (1)$$

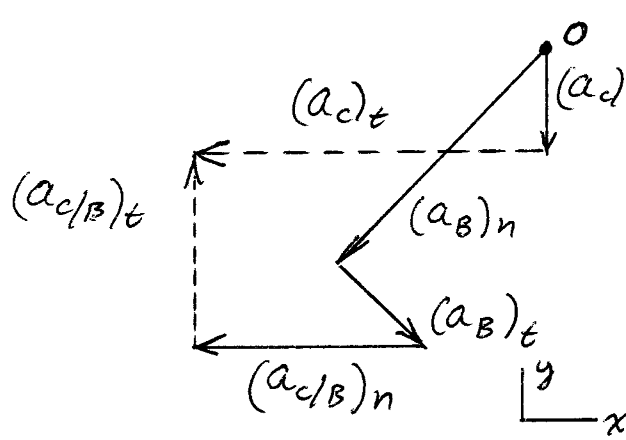
$$(a_c)_n = \omega_{CD}^2\, \overline{CD} = (2.5)^2(0.20) = 1.25 \ m/s^2$$

$$(a_B)_t = \alpha_{AB}\, \overline{AB} = (10)(0.1414) = 1.414 \ m/s^2$$

$$(a_B)_n = \omega_{AB}^2\, \overline{AB} = (5)^2(0.1414) = 3.535 \ m/s^2$$

$$(a_{c/B})_n = \omega_{BC}^2\, \overline{BC} = (5)^2(0.1) = 2.5 \ m/s^2$$

Vector diagram of (1):



$$(a_c)_t = \frac{(a_B)_n}{\sqrt{2}} - \frac{(a_B)_t}{\sqrt{2}} + (a_{c/B})_n$$

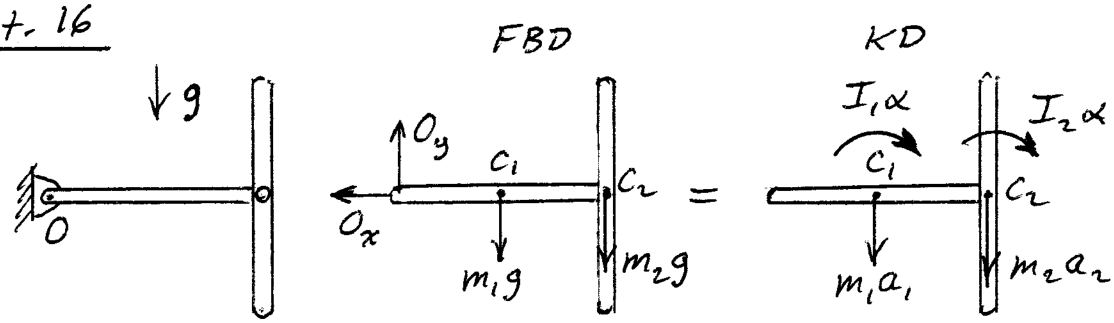
$$= \frac{3.535}{\sqrt{2}} - \frac{1.414}{\sqrt{2}} + 2.5$$

$$= 4 \ m/s^2$$

$$\underline{a}_c = -4\underline{i} - 1.25\underline{j}, \ \frac{m}{s^2}$$

Ans.

16.1



$$\curvearrowright + \sum M_o = m_1 a_1 \overline{OC_1} + I_1\alpha + m_2 a_2 \overline{OC_2} + I_2\alpha$$

$$a_1 = \overline{OC_1}\alpha, \quad a_2 = \overline{OC_2}\alpha$$

$$m_1 g\,\overline{OC_1} + m_2 g\,\overline{OC_2} = I_o\,\alpha$$

$$(20)(0.5) + (20)(1) = \left[\frac{1}{3}\left(\frac{20}{9.81}\right)1^2 + \frac{1}{12}\left(\frac{20}{9.81}\right)1^2 + \frac{20\times 1^2}{9.81}\right]\alpha$$

$$30 = 2.89\alpha$$

$$\alpha = 10.39 \text{ rad/s}^2 \ \curvearrowright$$

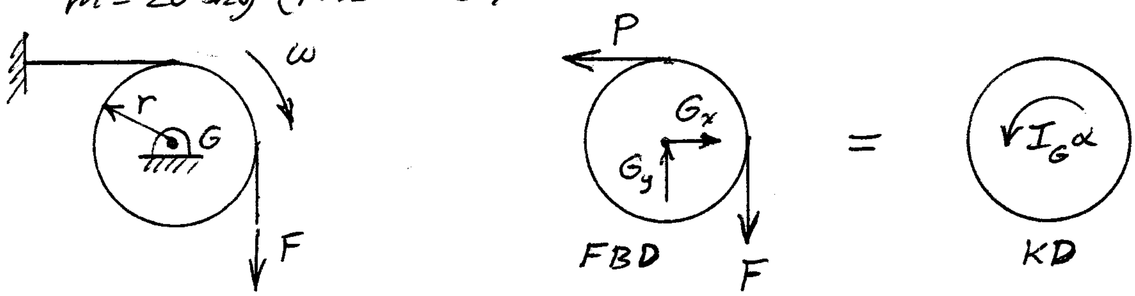
$$\sum F_y = -ma_1 - ma_2$$

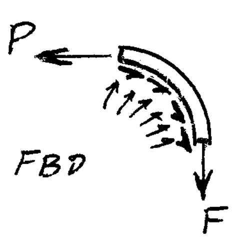
$$O_y - m_1 g - m_2 g = -ma_1 - ma_2$$

$$O_y = 20 + 20 - \frac{20}{9.81}(0.5\times 10.39 + 1\times 10.39)$$

$$= 8.24 \text{ N} \longleftarrow \text{Ans.}$$

16.2 $m = 20$ kg (MISSING)





$$+\circlearrowleft \sum M_G = I_G\,\alpha$$

$$(P - F)r = \frac{1}{2}mr^2\alpha$$

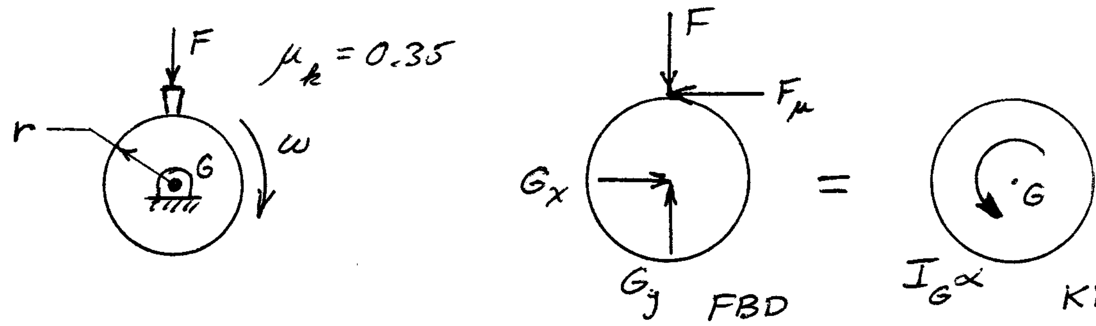
$$P = Fe^{\mu_k \pi/2} \quad \text{(p. 110, Handbook)}$$

$$\alpha = \frac{2F(e^{\mu_k \pi/2} - 1)}{mr} = \frac{2(350)(e^{0.35\pi/2} - 1)}{(20)(0.25)}$$

$$= 102.6 \text{ rad/s}^2$$

$$\omega = \omega_o - \alpha t, \quad \omega = 0 \text{ at } t = \frac{\omega_o}{\alpha} = \frac{500\times 2\pi}{60}\,\frac{1}{102.6} = 0.510 \text{ s} \longleftarrow \text{Ans.}$$

16.3



$$\overset{+}{\circlearrowleft}\ \Sigma M_G = I_G\,\alpha$$

$$F_\mu r = I_G\,\alpha \quad (1)$$

$$F_\mu = \mu_k F \quad (2)$$

(2) → (1):

$$\alpha = \frac{\mu_k F r}{I_G} \quad (3)$$

$\omega\,d\omega = \alpha\,d\theta \Rightarrow \omega^2 = \omega_o^2 - 2\alpha\theta$, $\omega = 0$ when $\theta = \frac{\pi}{2}$:

$$\alpha = \frac{\omega_o^2}{2\theta} = \left(\frac{\cancel{60}\times 2\pi}{\cancel{60}}\right)^2 \frac{1}{\pi}$$

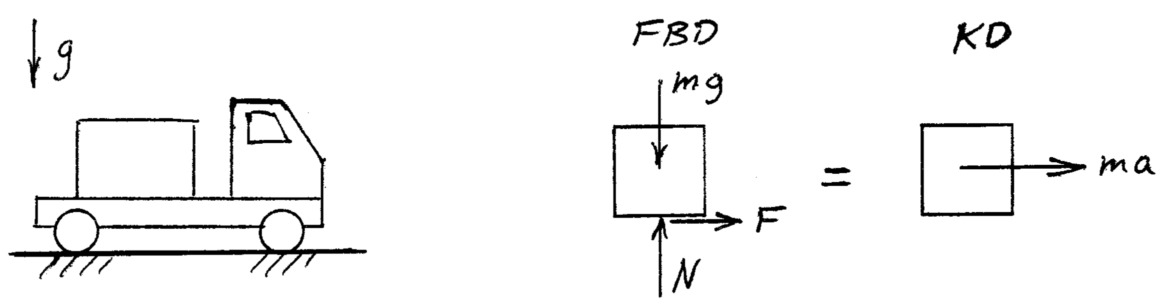
$$= 4\pi \quad (4)$$

(4) → (3)

$$F = \frac{I_G\,\alpha}{\mu_k r} = \frac{\frac{1}{2}(5)(0.35)^2\,4\pi}{(0.35)(0.35)}$$

$$= 31.4\,N \quad \leftarrow \text{Ans.}$$

16.4



$$\Sigma F_y = 0 \Rightarrow N = mg \quad (1)$$

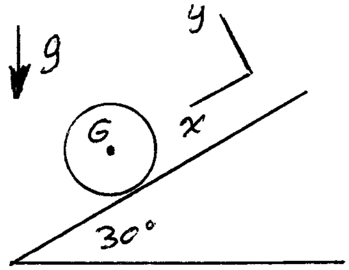
$$F = ma \quad (2)$$

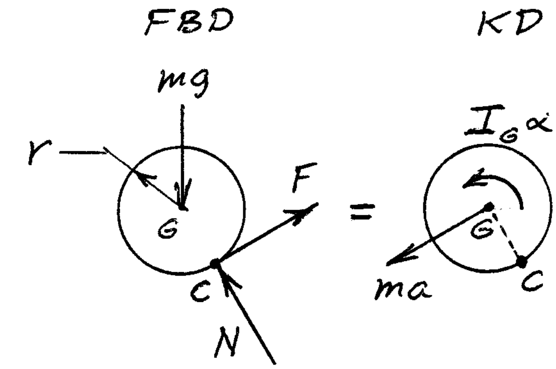
$$F = \mu_s N \quad (3)$$

From (1), (2) & (3): $a = \frac{1}{\cancel{m}}\mu_s \cancel{m} g = 0.4(9.81)$

$$= 3.92\,\frac{m}{s^2} \quad \leftarrow \text{Ans.}$$

16.5





$a = r\alpha$

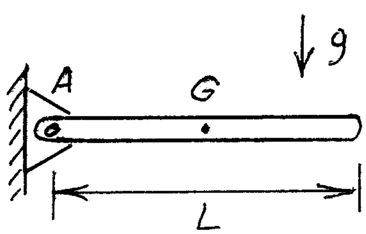
$$\overset{+}{\circlearrowleft} \; \Sigma M_c = mar + I_G\alpha$$

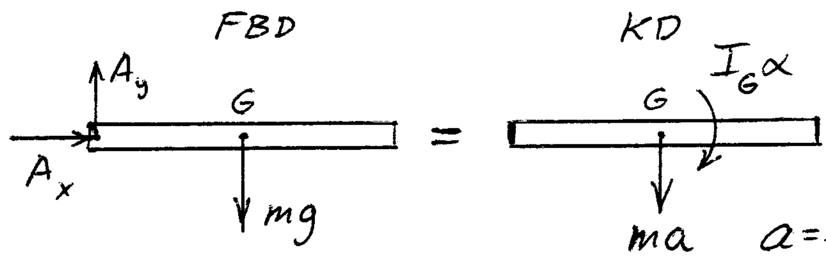
$$\cancel{m}gr\sin 30° = \cancel{m}ar + \frac{1}{2}\cancel{m}r^2\frac{a}{\cancel{r}}$$

$$a = \frac{2g\sin 30°}{3} = \frac{2(9.81)\sin 30°}{3}$$

$$a = 3.27\ m/s^2 \quad \longleftarrow \text{Ans.}$$

16.6





$a = \frac{L}{2}\alpha$

$$\Sigma F_y = -ma \Rightarrow A_y - mg = -ma \quad (1)$$

$$\overset{+}{\circlearrowright} \; \Sigma M_A = ma\frac{L}{2} + I_G\alpha$$

$$mg\frac{L}{2} = ma\frac{L}{2} + \frac{1}{12}mL^2\frac{2a}{L}$$

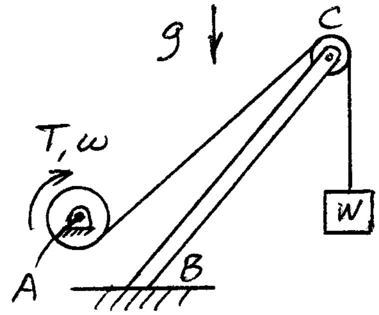
$$mg\frac{L}{2} = \frac{2}{3}maL \Rightarrow a = \frac{3}{4}g \quad (2)$$

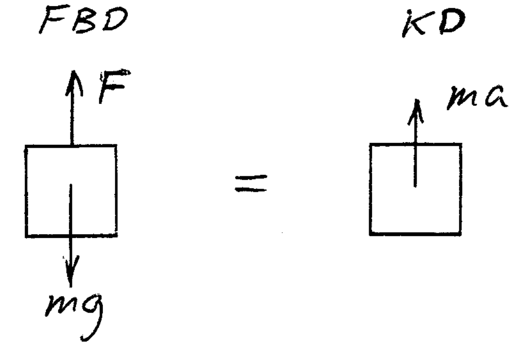
$$(2) \rightarrow (1) : \; A_y = mg - ma = mg - \frac{3}{4}mg = \frac{mg}{4}$$

$$= 10/4 = 2.5\ N \quad \longleftarrow \text{Ans.}$$

Chpt. 17

17.1

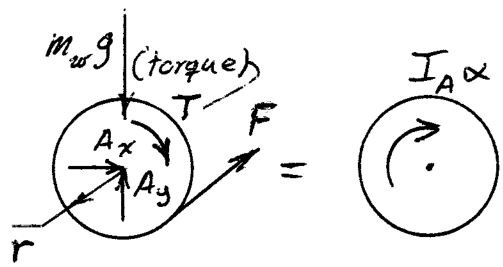




$$\Sigma F_y = ma$$

$$F - mg = ma$$

$$F = ma + mg$$
$$= \frac{10{,}000}{9.81}(1) + 10{,}000 = 11{,}019 \text{ N}$$



$$r\alpha = a$$

$$\circlearrowright + \ \Sigma M_A = I_A \alpha$$

$$T - Fr = I_A \alpha$$

$$T = Fr + I_A a/r \qquad \text{radius of gyration} = \sqrt{\frac{I_A}{m_w}}$$
$$= (11{,}019)(0.5) + (0.4)^2(600)(1)/0.5$$
$$= 5702 \text{ N}\cdot\text{m}$$

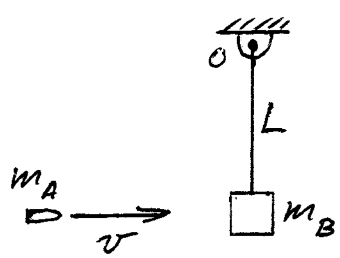
$$P = \text{Power} = T\omega = T\frac{v}{r} = 5702\,\frac{10/60}{0.5}$$

$$P = 1.90 \text{ kW} \quad \leftarrow \text{Ans.}$$

17.2 Above figure $\quad P = Fv \quad F = mg$

$$= mgv = (10{,}000)\left(\frac{10}{60}\right) = 1.67 \text{ kW} \quad \leftarrow \text{Ans}$$

17.3



Conservation of angular momentum about O:

$$m_A v \cancel{L} = (m_A + m_B)(\omega L)\cancel{L}$$

$$(0.035)(300) = (500.035)(0.5\omega)$$

$$\omega = 0.0420 \text{ rad/s} \quad \leftarrow \text{Ans}$$

17.4 Above figure.

Conservation of energy after impact:

$$0 = \Delta T + \Delta V_g \Longrightarrow \frac{1}{2}\cancel{(m_A + m_B)}(\omega L)^2 = \cancel{(m_A + m_B)}gL \times (1 - \cos 30°)$$

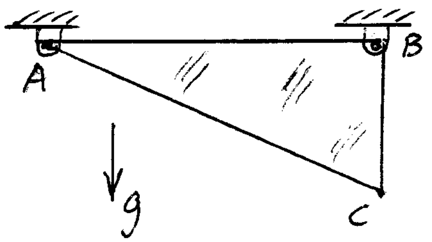
$$\omega^2 = \frac{(2)(9.81)(1 - \cos 30°)}{0.5}$$

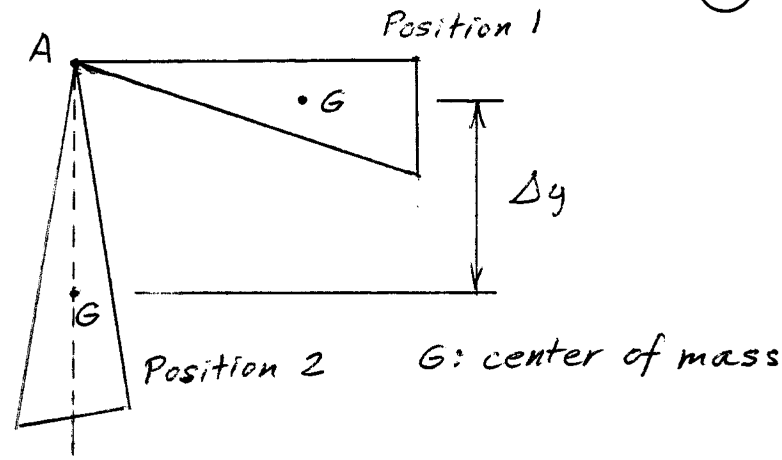
$$\omega = 2.29 \text{ rad/s}$$

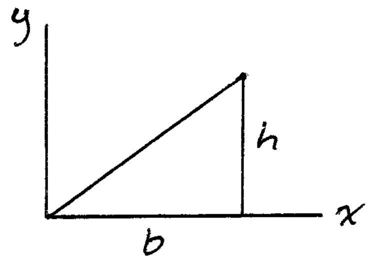
Conser. of momentum during impact

$$(0.035)v = (500.035)(0.5\omega) \Longrightarrow v = 16{,}379 \text{ m/s} \quad \leftarrow \text{Ans.}$$

17.5







Page 111, Handbook:

$$I_x = bh^3/12, \quad I_y = \frac{b^3 h}{4}$$

Therefore, $I_A = (I_x + I_y)\rho t$; ρ = density, t = thickness

$$= \frac{bh\rho t}{2}\left(\frac{h^2}{6} + \frac{b^2}{2}\right)$$

$$m = bh\rho t/2$$

$$I_A = m\left(\frac{h^2}{6} + \frac{b^2}{2}\right)$$

$$0 = \Delta T + \Delta V_g \Longrightarrow \frac{1}{2} I_A \omega^2 = mg\,\Delta y$$

$$\frac{1}{2}\cancel{m}\left(\frac{h^2}{6} + \frac{b^2}{2}\right)\omega^2 = \cancel{m}g\left\{\left[\left(\frac{2}{3}b\right)^2 + \left(\frac{h}{3}\right)^2\right]^{1/2} - \frac{h}{3}\right\}$$

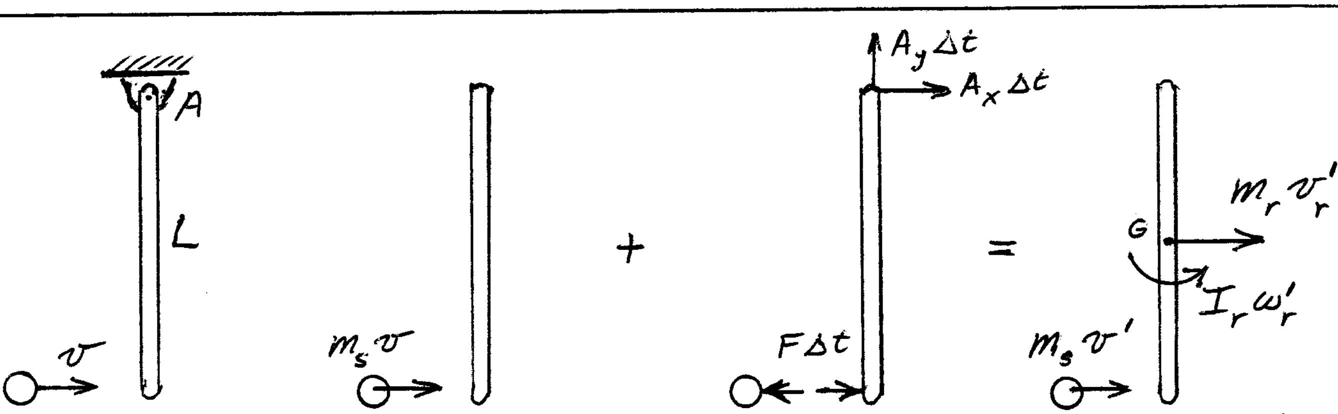
$$\frac{1}{2}\left(\frac{0.3^2}{6} + \frac{0.9^2}{2}\right)\omega^2 = 9.81\left\{\left[\left(\frac{2}{3}0.9\right)^2 + \left(\frac{0.3}{3}\right)^2\right]^{1/2} - \frac{0.3}{3}\right\}$$

$$0.21\,\omega^2 = 4.99$$

$$\omega^2 = 23.7$$

$$\omega = 4.87 \text{ rad/s} \longleftarrow \text{Ans.}$$

17.6



$$v_r' = \omega_r' \frac{L}{2}$$

Conservation of angular momentum about A:

$$m_s v L = m_s v' L + I_r \omega_r' + m_r v_r' \frac{L}{2}$$

$$m_s v L = m_s v' L + \frac{1}{12} m_r L^2 \frac{2 v_r'}{L} + m_r v_r' \frac{L}{2}$$

$$m_s v \cancel{L} = m_s v' \cancel{L} + \frac{2}{3} m_r v_r' \cancel{L} \quad (1)$$

$$e = \frac{2v_r' - v'}{v} \quad (2)$$

$$(1) \Rightarrow (1)(10) = (1)v' + \frac{2}{3}(10) v_r' \quad (3)$$

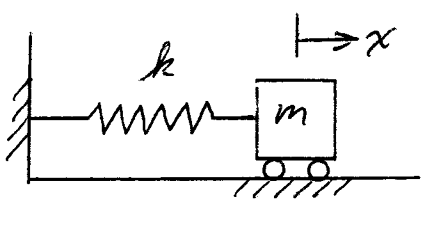
$$(2) \Rightarrow \quad (0.7)(10) = -v' + 2v_r' \quad (4)$$

$$(3) - \frac{10}{3} \times (4) \quad 10 - (7)\left(\frac{10}{3}\right) = \left(1 + \frac{10}{3}\right) v'$$

$$v' = -3.08 \ \frac{m}{s} \longleftarrow \text{Ans.}$$

Chpt. 19

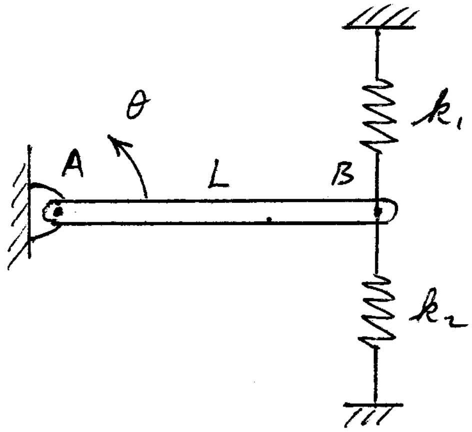
19.1

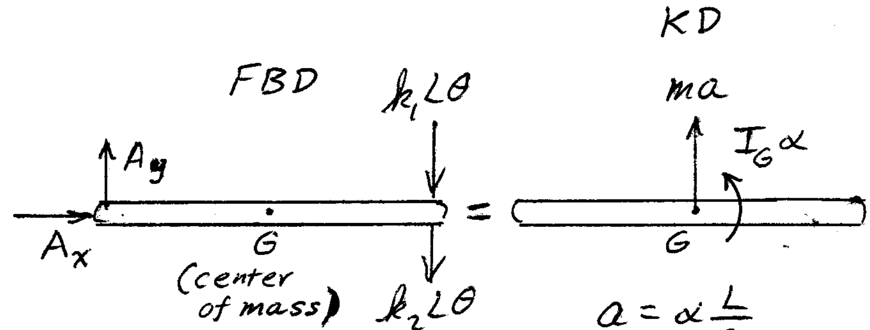


$k = 800\ N/m$
$m = 6\ kg$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{800}{6}} = 11.55\ \frac{rad}{s} \quad \leftarrow \text{Ans.}$$

19.2

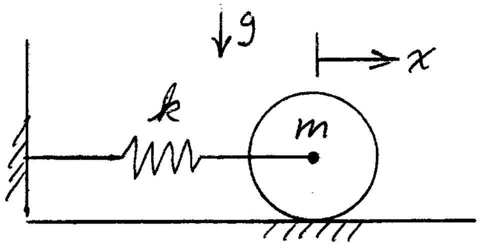


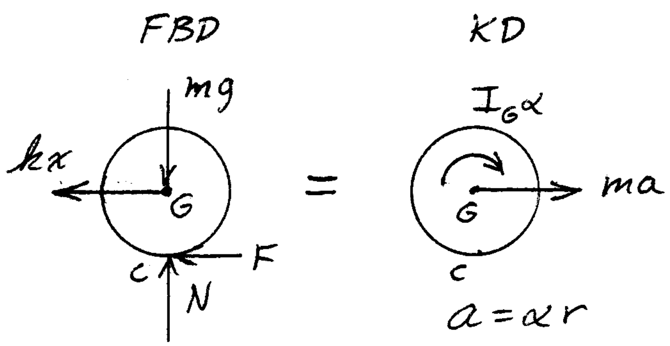


$a = \alpha \frac{L}{2}$

$$\circlearrowleft \Sigma M_A = ma\frac{L}{2} + I_G\alpha$$
$$-(k_1+k_2)L^2\theta = m\alpha\left(\frac{L}{2}\right)^2 + I_G\alpha$$
$$= I_A\alpha$$
$$I_A\ddot{\theta} + (k_1+k_2)L^2\theta = 0$$
$$\omega_n = \sqrt{\frac{(k_1+k_2)L^2}{I_A}} = \sqrt{\frac{(k_1+k_2)L^2}{\frac{1}{3}mL^2}} = \sqrt{\frac{1600\times 3}{10}}$$
$$= 21.9\ rad/s$$
$$\omega_n = \frac{2\pi}{\tau} \Longrightarrow \tau = 0.287\ s \quad \leftarrow \text{Ans.}$$

19.3





$a = \alpha r$

$$\circlearrowleft \Sigma M_c = mar + I_G\alpha$$
$$-kxr = m\ddot{x}r + \frac{1}{2}mr^2\frac{\ddot{x}}{r}$$

(Cont'd next page)

$$\frac{3}{2} m\ddot{x} + kx = 0$$

$$\omega_n = \sqrt{\frac{k}{\frac{3}{2}m}} = \sqrt{\frac{850}{\frac{3}{2}10}} = 7.53 \ \frac{rad}{s}$$

$$x = 0.02 \cos(\omega_n t)$$

$$\dot{x} = 0.02\,\omega_n(-\sin(\omega_n t))$$

$$|\dot{x}|_{max} = 0.02\,\omega_n = 0.151 \ \frac{m}{s} \longleftarrow \text{Ans.}$$

19.4 Above figure $\omega_n = \frac{2\pi}{\tau} \Rightarrow \tau = 0.835\,s \longleftarrow$ Ans.

19.5 Figure on top of page 18

$$x = 0.030 \cos(\omega_n t)$$

$$\dot{x} = -0.03\,\omega_n \sin(\omega_n t)$$

$$|\dot{x}|_{max} = 0.03\,\omega_n = 0.346 \ m/s \longleftarrow \text{Ans.}$$

19.6

$$\ddot{x} = -0.03\,\omega_n^2 \cos(\omega_n t)$$

$$|\ddot{x}|_{max} = 0.03\,\omega_n^2 = 4 \ m/s^2 \longleftarrow \text{Ans.}$$