① (a)

$$\frac{2-3i}{1-2i}\cdot\frac{1+2i}{1+2i} = \frac{2+4i-3i-6i^2}{1-4i^2}$$

$$= \frac{8+i}{5} = \frac{8}{5}+\frac{1}{5}i$$

① (b) $$e^{-2+\pi i} = e^{-2}\left[\underbrace{\cos(\pi)}_{-1} + i\underbrace{\sin(\pi)}_{0}\right] = -e^{-2}$$

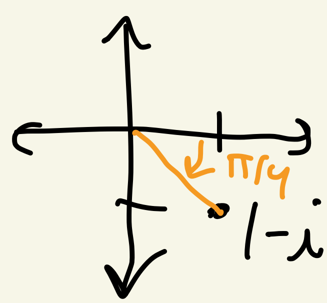
① (c)

$$\cos(2\pi+i) = \frac{e^{i(2\pi+i)} + e^{-i(2\pi+i)}}{2} = \frac{1}{2}\left[e^{2\pi i-1} + e^{-i2\pi+1}\right]$$

$$= \frac{1}{2}\left[e^{-1}\left[\underbrace{\cos(2\pi)}_{1} + i\underbrace{\sin(2\pi)}_{0}\right] + e\left[\underbrace{\cos(-2\pi)}_{1} + i\underbrace{\sin(-2\pi)}_{0}\right]\right]$$

$$= \frac{1}{2}\left[e^{-1}+e\right]$$

① (d)



$$(1-i)^i = e^{i\log(1-i)} = e^{i[\ln(\sqrt{2})+i(-\pi/4)]}$$

$$= e^{i\ln(\sqrt{2})+\pi/4}$$

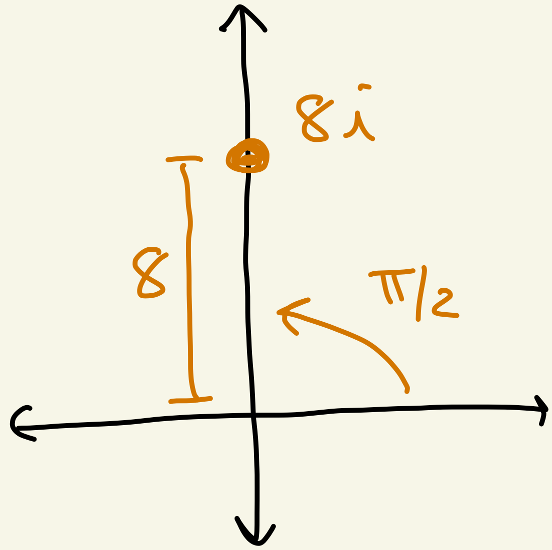
$$= e^{\pi/4}\left[\cos(\ln(\sqrt{2})) + i\sin(\ln(\sqrt{2}))\right]$$

$$= e^{\pi/4}\cos(\ln(\sqrt{2})) + i\,e^{\pi/4}\sin(\ln(\sqrt{2}))$$

(2) $z^3 - 8i = 0$

$z^3 = 8i$

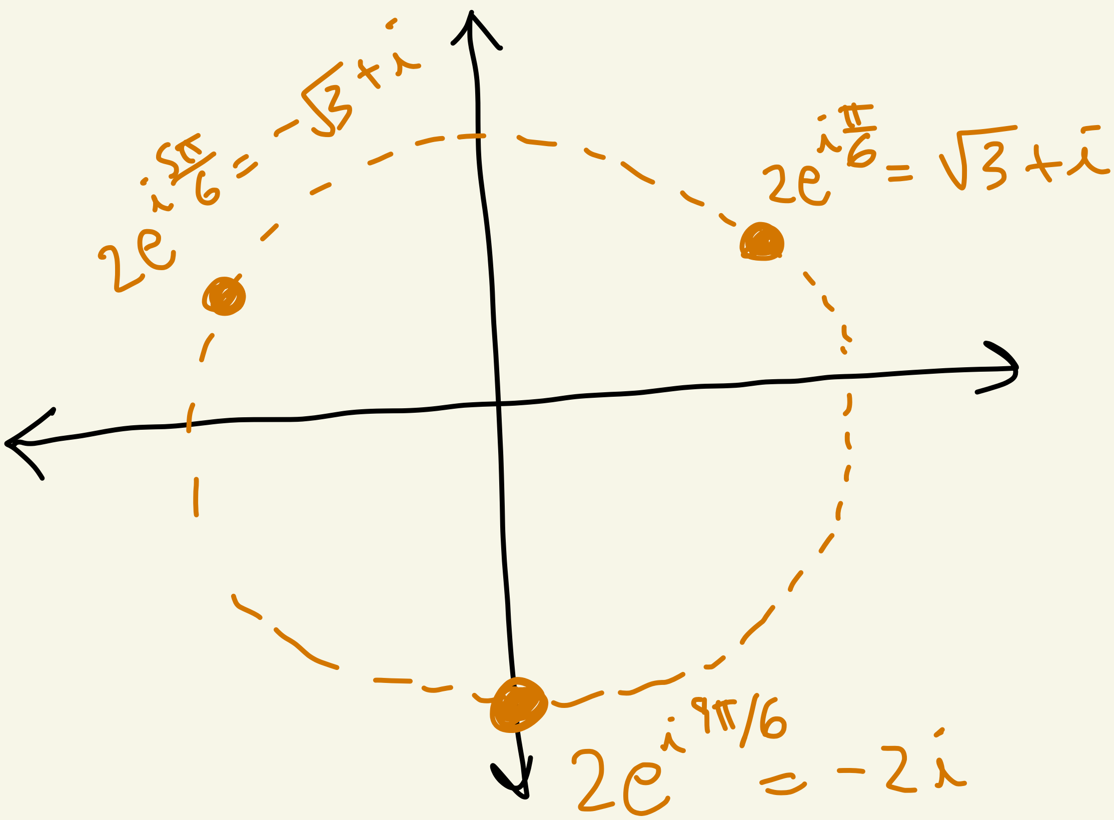
$$z_k = 8^{1/3} e^{\left(\frac{\pi/2}{3} + \frac{2\pi k}{3}\right)i}, \quad k = 0, 1, 2$$



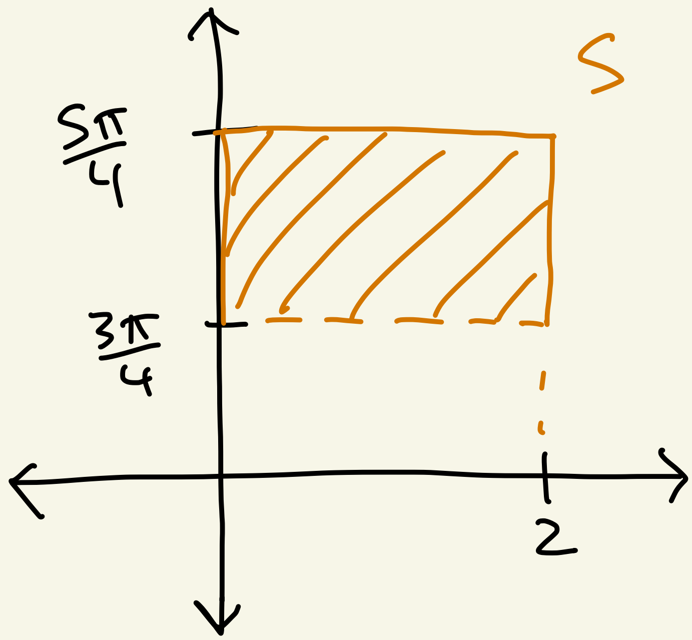
$$z_0 = 2e^{i\pi/6} = 2\left[\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right] = 2\left[\frac{\sqrt{3}}{2} + i\frac{1}{2}\right] = \sqrt{3} + i$$

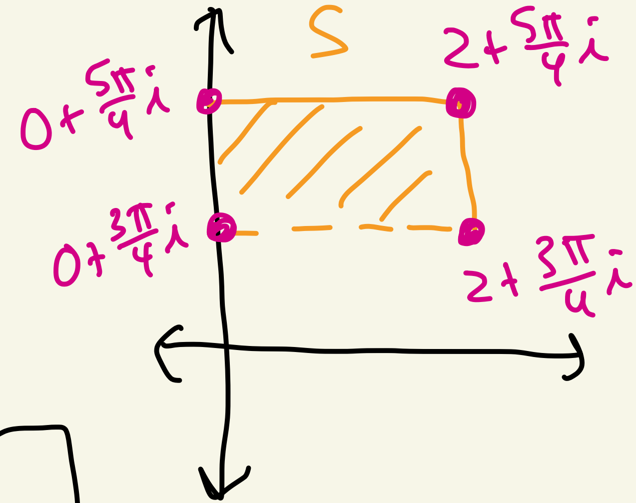
$$z_1 = 2e^{i5\pi/6} = 2\left[\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right] = 2\left[-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right] = -\sqrt{3} + i$$

$$z_2 = 2e^{i9\pi/6} = 2\left[\cos\left(\frac{9\pi}{6}\right) + i\sin\left(\frac{9\pi}{6}\right)\right] = 2\left[0 - i\right] = -2i$$

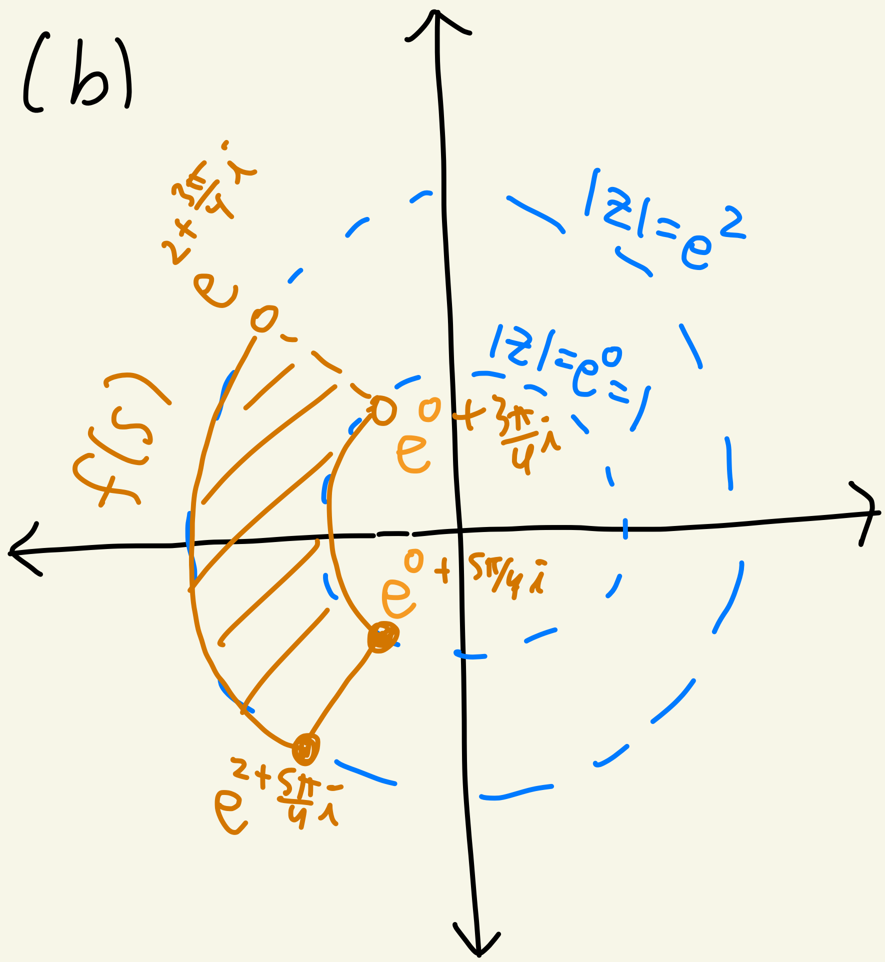


③ (a)

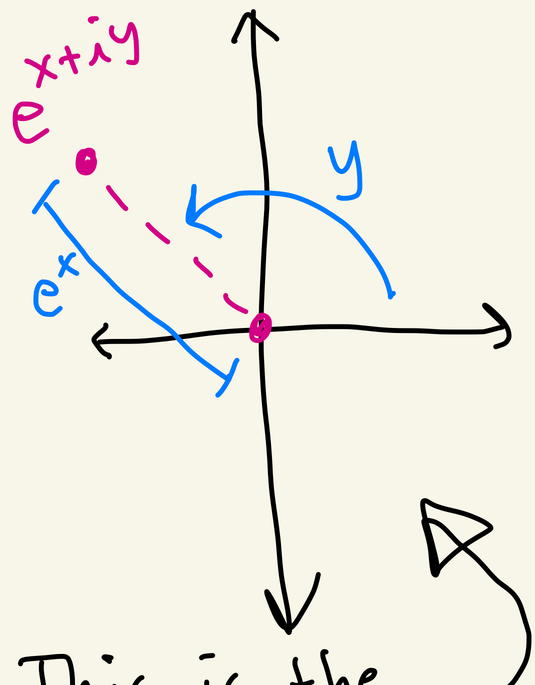




(b)



Recall:



This is the main thing to know to be able to graph e^z

A or B

(A) See HW 1 #8(b,d)

(B) See HW 2 #11

C or D

(c) Let's show $A \cup B$ is open.

We assume $A \neq \phi$ and $B \neq \phi$.

Then $A \cup B \neq \phi$.

Let $z \in A \cup B$.

Then $z \in A$ or $z \in B$.

<u>case 1:</u> Suppose $z \in A$.

Since A is open, z is an interior pt of A.

So, $\exists r_a > 0$ where $D(z; r_a) \subseteq A$.

Then, $D(z; r_a) \subseteq A \subseteq A \cup B$.

So, z is an interior pt of $A \cup B$.

<u>case 2:</u> Suppose $z \in B$.

Since B is open, z is an interior pt of B.

So, $\exists r_b > 0$ where $D(z; r_b) \subseteq B$.

Then, $D(z; r_b) \subseteq B \subseteq A \cup B$.

So, z is an interior pt of $A \cup B$.

In either case z is an interior pt of $A \cup B$.

So all of $A \cup B$'s elements are interior points

Hence $A \cup B$ is open

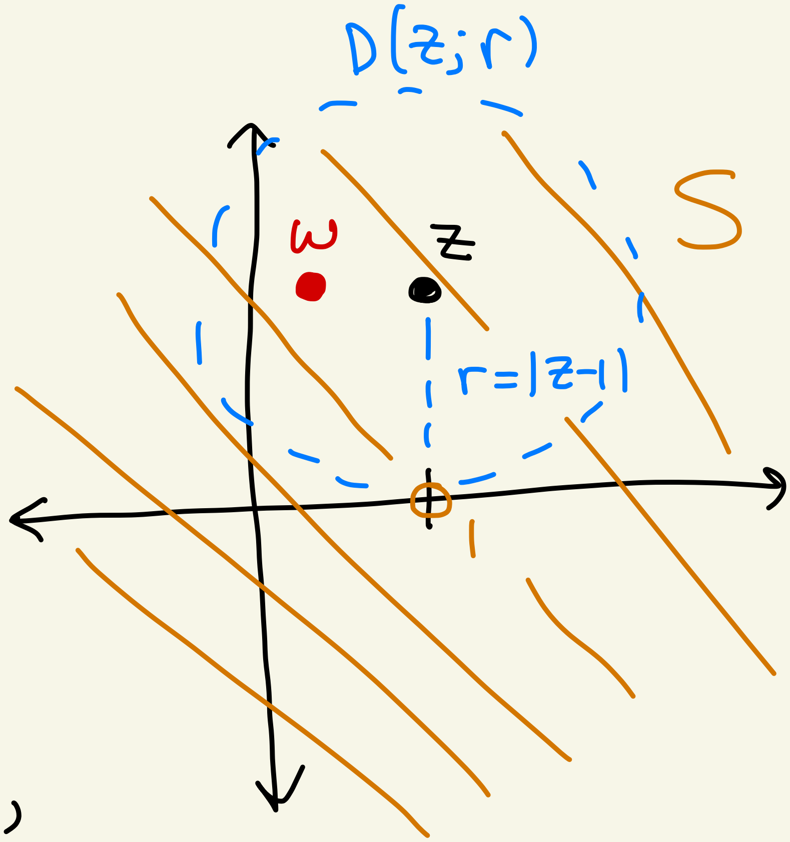
(D) Same proof as $\mathbb{C}-\{0\}$ in class with 1 exchanged for 0, or same as HW 3 #3(e).

Let $z \in \mathbb{C}-\{1\}$.

Let $r = |z-1|$

Consider $D(z;r)$.

We must show $D(z;r) \subseteq S$.



Let $w \in D(z;r)$.

Then, $|w-z| < r$.

We must show $w \in S$, that is $w \neq 1$.

Suppose $w = 1$.

Then $|z-1| = |z-w| < r = |z-1|$

So, $|z-1| < |z-1|$ which can't happen.

Thus, $w \neq 1$ and $w \in S$.

So, $D(z;r) \subseteq S$.

So, z is an interior pt of S.

So, S is open.