<u>Test 1 study guide</u>

The test will consist of computations/specific examples and proofs. There will be at least one proof that is directly from the homework on the test. There may be true or false (i.e., if true prove it, if false give a counterexample).

Here is an outline to study from. I've broken down the topics and put examples of each topic from the homework.

<u>Part 1 – Computations</u>

- Computing group products, group tables, and inverses in the various groups.
  (See HW 1.1,1.2,1.3, and Math 4550 HW 1 and HW 5)

- Computing the order of an element. Finding generators of groups.
  (See HW 1.1 and 1.2; HW 2.3 – A; Math 4550 HW 2 and HW 5; Math 4550 HW 7 - #1)

- Computing the cyclic subgroup generated by an element. Finding all subgroups of a cyclic group.
  (See HW 2.3; Math 4550 HW 2; Math 4550 HW 4 - #8; Math 4550 HW 7 - #4)

- Finding or verifying that a group has a non-cyclic subgroup. Testing if a group is cyclic or not.
  (See HW 2.3 #11,12)

- Determining whether a specific subset of a group is a subgroup or not.
  (See HW 2.1 - #1,3,14; 2.2 and Math 4550 HW 2 - #1,4,6)

- Is a function a homomorphism or not?
  (See Math 4550 HW 3 - #1)

- Finding all homomorphisms $f : G \to H$ where G is a cyclic group.
  (See Math 4550 HW 4 - #3,5,6,15)

<u>Part 2 – Proofs</u>

Take a look at the homework proofs and ones we did in class for studying. Here are some ones to do first to prioritize your studying.

- Determining whether or not a set with an operation is a group or not.
  (See HW 1.1 – # 5,8 and Math 4550 HW 1 - #4,5,7)

- Determining whether some set is a subgroup or not.
  (See HW 2.1 - #10,11,12; HW 2.2 - #6,11, and Math 4550 HW 2 – #13,14,15,16)

- Determining whether or not two groups are isomorphic.
  (See HW 1.6 - #4,5,6; HW 2.3 – 13; Math 4550 HW 4 - #16)

- Determining whether or not an operation is a group action.
  (See HW 1.7 - #14,15,16)

- Proofs relating to the centralizer, normalizer, center
  (See HW 2.2 - #2,6,11)

- General proofs about groups and homomorphisms.
  (For example, HW 1.1 #25, 28; 1.6 #2,3,13; Math 4550 HW 3 - #3; Math 4550 HW 4 - #13)

- Some specific proofs from class or HW:

  1. The identity element of a group is unique. (in class / Sam's notes)

  2. Inverses in a group are unique. (in class / Sam's notes)

  3. Let f : G --> G' be a homomorphism and H be a subgroup of G and H' be a subgroup of G'. Let 1_G and 1_G' be the identity elements of G and G'. Then:

     (i) f(1_G) = f(1_G') (in class)
     (ii) f(x^(-1)) = (f(x))^(-1) for all x in G (in class)
     (iii) ker(f) is a subgroup of G (in class)
     (iv) If x has order n, then f(x) has order dividing n. (4550 – HW 3 #5)

- For any group G and element x in G, the set <x> is a subgroup of G. (in class)
- Every cyclic group is abelian (in class)