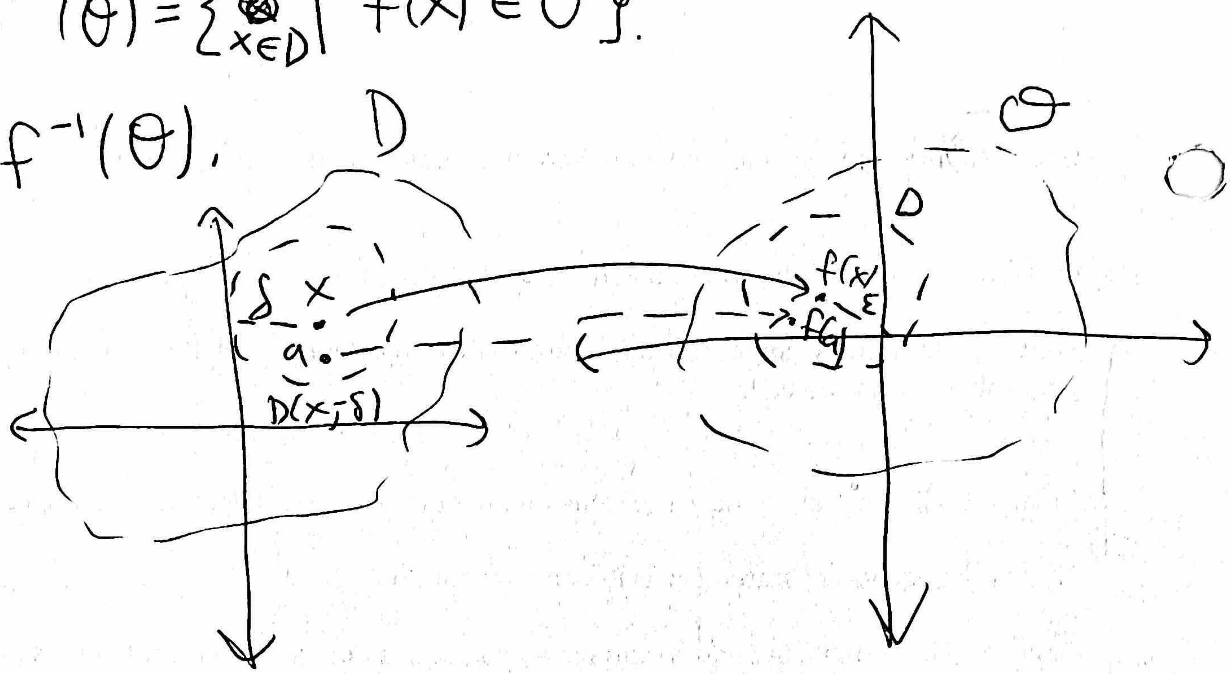
Recall: If $f : D \to \mathbb{C}$ is continuous on an open set D and $\mathcal{O} \subseteq \mathbb{C}$ is an open set, then $f^{-1}(\mathcal{O}) \subseteq D$ is open in $\mathbb{C}$.

$f^{-1}(\Theta) = \{x \in D \mid f(x) \in \Theta\}$.

<u>Proof:</u> Let $x \in f^{-1}(\Theta)$.



Then $f(x) \in \Theta$.

Since Θ is open, there exists $\varepsilon > 0$ so that $\mathcal{D} = D(f(x);\varepsilon) \subseteq \Theta$.

Since f is continuous, there exists $\delta > 0$ so that if $\underbrace{|a-x|<\delta}_{\text{same as: } a \in D(x;\delta)}$

then $\underbrace{|f(a)-f(x)|<\varepsilon}_{\text{Same as } f(a)\in D(f(x);\varepsilon)}$.

Thus, all $a \in D(x;\delta)$ satisfy $f(a) \in \mathcal{D}$.

Thus all $a \in D(x;\delta)$ satisfy $\underbrace{f(a) \in \Theta}_{\text{Same as } a \in f^{-1}(\Theta)}$.

So, $D(x;\delta) \subseteq f^{-1}(\Theta)$.

Thus x is an interior pt of $f^{-1}(\Theta)$ and $f^{-1}(\Theta)$ is open. □