/ Solutions

Quiz 2, Math 209, Winter 2007 **Name:**___________________

Directions: Show ALL of your work to get credit. If you leave something out, then you may be penalized. No calculators. Good luck!

IMPORTANT: There is a second side to this quiz. It has 4 problems.

1. [5 points] Compute $L(x, y)$ of the function

$$f(x,y) = \frac{x}{y}$$

at the point $(6, 3)$.

$$L(x,y) = f(6,3) + f_x(6,3)(x-6) + f_y(6,3)(y-3)$$

$$f_x(x,y) = \frac{1}{y} \quad \text{and} \quad f_y(x,y) = -\frac{x}{y^2}$$

$$L(x,y) = 2 + \frac{1}{3}(x-6) + \left(-\frac{2}{3}\right)(y-3)$$

$$= \frac{1}{3}x - \frac{2}{3}y + 2$$

2. [5 points] Let

$$z = x^2 + xy^3, \quad x = uv^2 + w^3, \quad y = u + ve^w.$$

Find $\frac{\partial z}{\partial v}$ where $u = 2$, $v = 1$, and $w = 0$.

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial v}$$

$$= (2x + y^3)(2uv) + (3xy^2)(e^w)$$

$$= (2 \cdot 2 + 3^3)(2 \cdot 2) + (3 \cdot 2 \cdot 3^2)$$

$$= (4+27) \cdot 4 + 27 \cdot 2$$

$$= 124 + 54 = 178$$

$x = 2 \cdot 1 = 2$
$y = 2 + 1 = 3$

3. [5 points] Determine the set of points at which the function f is continuous where

$$f(x,y) = \begin{cases} \frac{8x^2y^2}{x^4+y^4} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Make sure to give reasons why.

If we let $y=x$, then

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,x)\to(0,0)} \frac{8x^4}{x^4+x^4} = \lim_{x\to 0} 4 = 4 \neq 0 = f(0,0).$$

So, $f(x,y)$ is not continuous at $(0,0)$.

It is continuous everywhere else since it equals $\frac{8x^2y^2}{x^4+y^4}$ and $x^4+y^4>0$ if $(x,y)\neq(0,0)$.

So, the answer is all (x,y) except at $(x,y)=(0,0)$.

4. [5 points] Use the definition of partial derivatives as limits to find $f_x(x,y)$ where

$$f(x,y) = y^2 - xy + 2x^2$$

$$f_x(x,y) = \lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h} =$$

$$= \lim_{h\to 0} \frac{y^2-xy-hy+2x^2+4xh+2h^2-y^2+xy-2x^2}{h}$$

$$= \lim_{h\to 0} (-y+4x+2h) = -y+4x$$