Quiz 6, Math 209, Winter 2007 **Name:** Solutions

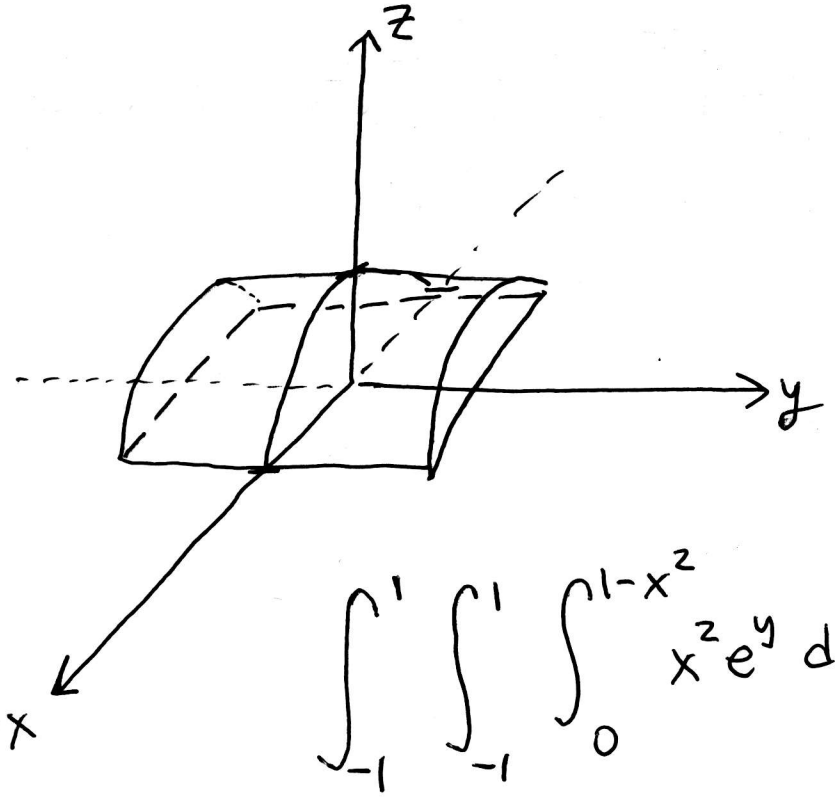
Directions: Show ALL of your work to get credit. If you leave something out, then you may be penalized. No calculators. Good luck!

IMPORTANT: This quiz has THREE problems.

1. [6.5 points] Setup the integral

$$\iiint_E x^2 e^y \, dV$$

but DO NOT EVALUATE it, where E is the solid that is bounded by the cylinder $z = 1 - x^2$ and the planes $z = 0$, $y = 1$, and $y = -1$.

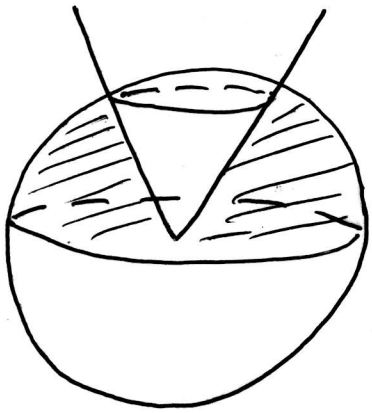


$$\int_{-1}^{1}\int_{-1}^{1}\int_{0}^{1-x^2} x^2 e^y \, dz\, dx\, dy$$

2. [6.5 points] Setup an integral, but DO NOT EVALUATE it, to find the volume of the solid E that lies within the sphere $x^2+y^2+z^2=1$, above the xy-plane, and below the cone $z=\sqrt{x^2+y^2}$.

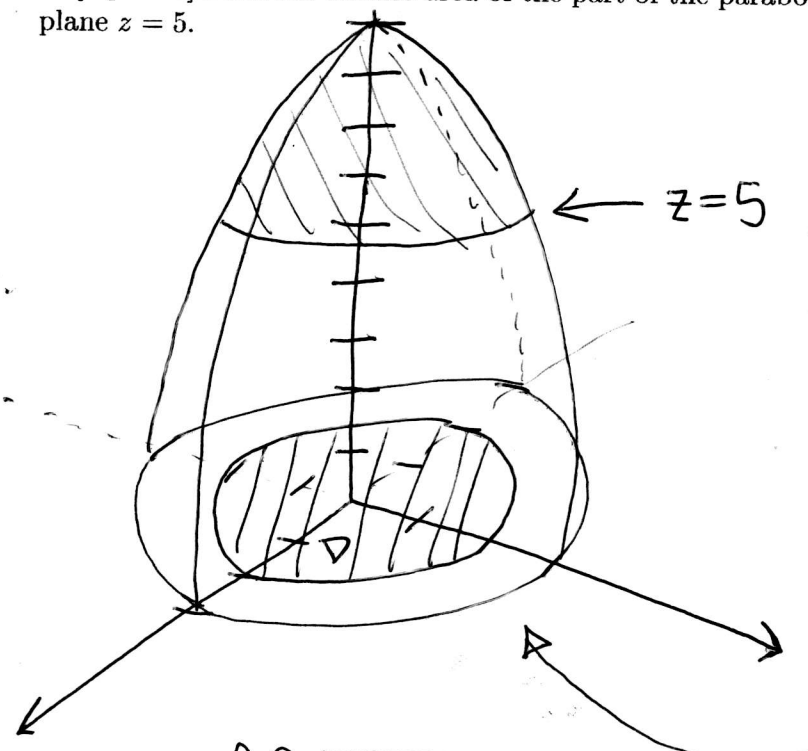
Recall the following formulas:

$$\begin{aligned} x &= \rho\sin(\phi)\cos(\theta) \\ y &= \rho\sin(\phi)\sin(\theta) \\ z &= \rho\cos(\phi) \\ \rho^2 &= x^2+y^2+z^2 \\ dV &= \rho^2\sin(\phi)\,d\rho\,d\theta\,d\phi \end{aligned}$$



$$V=\int_0^{2\pi}\int_0^{1}\int_{\pi/4}^{\pi/2}\rho^2\sin(\phi)\,d\phi\,d\rho\,d\theta$$

3. [7 points] Find the surface area of the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the plane $z = 5$.



$z=5$ gives

$5 = 9 - x^2 - y^2$

So, $x^2 + y^2 = 4$.

So, integrate over inner circle.

$$S = \iint_D \sqrt{(-2x)^2 + (-2y)^2 + 1}\, dA = \iint_D \sqrt{4x^2 + 4y^2 + 1}\, dA$$

$$S = \int_0^{2\pi} \int_0^2 \sqrt{4r^2 + 1}\; r\, dr\, d\theta$$

$$= \frac{1}{8} \int_0^{2\pi} \int_1^{17} \sqrt{u}\, du\, d\theta$$

$u = 4r^2 + 1$

$du = 8r\, dr$

$$= \frac{1}{8} \int_0^{2\pi} \frac{2}{3} u^{3/2} \Big|_1^{17} d\theta$$

$$= \frac{1}{8} \int_0^{2\pi} \left[\frac{2}{3} 17\sqrt{17} - \frac{2}{3}(1) \right] d\theta$$

$$= \frac{2\pi}{8}\left(\frac{2}{3}\right)\left[17\sqrt{17} - 1\right] = \frac{\pi}{6}\left[17\sqrt{17} - 1\right]$$